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Essays in Macroeconomics

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Declaration This thesis titled, "Essays in Macroeconomics" and the work presented in it are my own except where it contains work based on collaborative research. The first chapter is the outcome of long lasting discussions with my supervisor Roberto Pancrazi who also contributed to the drafting of the paper. This thesis also includes joint work with Giulio Trigilia and Herakles Polemarchakis (the 2d chapter) as well as joint work with Giovanni Callegari and Niccolò Battistini (the 3d chapter). The thesis has not been submitted for a degree at any other university.

Abstract

This dissertation consists in three essays that share as a common motivation the study of economic policy in models of general equilibrium. A brief summary is presented below.

Ch.1 Perfectly competitive markets for sovereign bonds that carry default risk are characterized by what we label as the *non-exclusivity externality*: when the ownership of debt is anonymous and dispersed, the equilibrium price of new debt on the primary market might be too low to avoid default, even though preventing default would be in the interest of existing creditors. We show that a policy maker can solve this externality ex-post, but the benefit ex-ante depends crucially on the ability of the government to tax bond holdings.

Ch.2 This chapter studies a stylised RBC in a model where credit markets are subject to adverse selection. On the normative side we show that competitive equilibria are constrained suboptimal when hidden information problems are severe. On the positive side, we show that a purely informational shock that shifts the distribution of firms subject to asymmetric information, might be an important driver of business cycle fluctuations.

Ch.3 We analyse how different types of monetary policy regimes and constraints affect debt limits, defined as the maximum level of debt an economy can service. We find that a more reactive monetary policy stance generally raises the debt limit, by reducing the inefficiencies linked to inflation fluctuations. On the other hand the ZLB negatively affects debt limits by amplifying the fall in tax revenue and muting the decrease in the interest rate. We then show that making debt limits endogenous to monetary policy considerations has important consequences on the transmission of fiscal policy shocks.

To Berta and Anastasia

Chapter 1

Non-exclusivity externality in Sovereign bond markets

1.1 Introduction

The recent European sovereign crisis has called for unconventional and unprecedented policy responses to overcome a long period of fiscal distress, financial turmoil, and uncertainty. With the introduction of various programs and policies,[1] the European Central Bank (henceforth ECB) aimed to inject liquidity to banks and to intervene in sovereign debt markets with the explicit goal of stopping the dangerous spiral in spreads on sovereign bonds and protecting bondholders' interest. The sovereign crisis has generated discussion among policymakers and researchers over the merits and risks of these policies.[2]

Given the relevance of the European sovereign crisis, the recent literature has investigated possible causes of market failures and explored possible policy solutions. One strand of the literature has rationalized the scope of intervention policies in the bond market with the aim of preventing a self-fulfilling crisis. This line of research has been pioneered by Diamond and Dybvig (1983)

[1]For example the European Financial Stability Facility (EFSF), the European Financial Stabilisation Mechanism (EFSM), and the European Stability Mechanism (ESM).

[2]Examples of work that evaluate the effects of ECB policies are Merler, Pisani-Ferry, et al. (2012), De Pooter, Martin, and Pruitt (2012), Eser and Schwaab (2012), Altavilla, Giannone, and Lenza (2014), Krishnamurthy, Nagel, and Vissing-Jorgensen (2015), Falagiarda and Reitz (2015), and Szczerbowicz et al. (2015), among others.

and includes works by Gertler and Kiyotaki (2015), Lorenzoni and Werning (2014), Broner et al. (2014), Corsetti, Dedola, et al. (2016), De Grauwe and Ji (2013). In a nutshell, the basic idea is that the economy can be stuck in a bad equilibrium caused by investors' pessimistic expectations and policy intervention is able to revert the economy to a good equilibrium by acting as a lender of the last resort and providing deposit insurance. Another strand of the literature has explored the degree of inefficiency created by the debt dilution problem; as discussed in Bolton and Jeanne (2009), Chatterjee and Eyigungor (2013), Hatchondo, Martinez, and Sosa-Padilla (2016), the debt-dilution problem is caused by the government's lack of commitment not to decrease the value of debt issued in the past by issuing new debt.

In this paper we propose an alternative and novel reason that justifies policy interventions in sovereign bond markets, by showing that perfectly competitive markets for sovereign bonds that carry default risk are characterized by an additional externality. In fact, in a competitive market, in which the ownership of debt is anonymous and dispersed, the equilibrium price of new debt on the primary market might be too low to avoid default, even though preventing default would be in the interest of existing creditors. For example, if that bond is issued by a sovereign country that is on the verge of default, the price would intrinsically be low. Borrowing at a high interest rate will, in turn, speed up that country's default decision. This prospect is unwelcome by existing bondholders since they would like, if they could, to offer better borrowing conditions to the troubled economy in order to delay default. In a competitive market, however, existing bondholders are not able to affect the bond price. This externality, which we label as *non-exclusivity externality*, arises because of the existence of a large number of atomistic lenders. In fact, if there was only one lender in the market this externality would disappear. As pointed out by Hellwig (1977) in a more general setting, credit granted to a single borrower is not a homogeneous good, since later loans affect the return on earlier loans. Therefore an existing creditor is able to extend new loans at conditions that nobody else would be willing to accept. At the endogenous debt limit, the expected present value of returns to new lenders becomes negative, even though the value for existing lenders remains positive, and additional loans are profitable only because they improve previous loans. An important consequence of this externality is that the endogenous debt limit for the sovereign economy that borrows in competitive markets is much lower than the one implied by eliminating the externality.

Three are the contributions of our paper. First, we highlight the presence of the *non-exclusivity externality* in a standard incomplete market for sovereign bonds. Second, we define a policy intervention that addresses this externality and we highlight the ex-post equilibrium properties. Third, we investigate the ex-ante properties of the equilibrium that includes the intervention, with particular emphasis on the welfare implications. The main result is that while the policy intervention is ex-post Pareto improving, its ex-ante welfare implications crucially depend on the fiscal policy used to finance the intervention. Nevertheless, we show that there is a set of fiscal policies that obtain an ex-ante Pareto improvement.

Let us first summarize our framework. There are three agents in the economy: a peripheral country, international investors, and a policy maker. *(i)* A peripheral country starts in a recessionary state and finances its consumption streams issuing long-term bonds. Stochastically, the exogenous income eventually jumps to a good state, in which it will remain forever. However, if the recession is long lasting the government might accumulate too much debt and it will optimally default. In this case, it stops interest repayments to bondholders and it will remain in financial autarky until the recession is over. At that point it will repay the investors after a renegotiation of the debt burden. Hence, the problem of the peripheral economy is standard and it is in the same spirit of the endogenous default framework as in Eaton and Gersovitz (1981). *(ii)* Risk neutral international investors buy sovereign bonds in a perfectively competitive market and bear the risk of a loss due to the government's insolvency in case of default. Their optimal decision results in an equilibrium bond price. *(iii)* Finally, a policy maker might decide to implement a policy, which will be described in details below, which offers to the peripheral government the possibility to borrow at more favourable conditions, in order to extend interest repayment to bond holders. The intervention is budget-balanced and financed by levying taxes to bondholders.

The policy intervention in our framework works as follows. When default comes, the policy maker can solve the *non-exclusivity externality* by subsidizing new lending to the small open economy. The size of the subsidy, and therefore the bond price after the policy implementation, is chosen so that the peripheral country is indifferent between defaulting and continuing to borrow at the policy price. As a consequence, the small open economy can borrow at better conditions than the one offered by competitive investors absent the policy. This way the government can delay default and continue to repay bondholders. We also assume that the cost of the policy is financed by taxes levied to investors. Therefore, the policy maker runs a balanced budget. We show that this type of intervention is, at the moment of its implementation, Pareto improving. In fact, investors are better off because the benefits obtained by the increased probability to receive a full repayment of their existing loans exceeds the expected loss from financing the intervention. Hence, the policy is able to eliminate the *non-exclusivity externality* that characterizes the competitive bond market. In addition, we show that the policy intervention has always a limited duration. In fact, after the policy starts, if the recession continues to last, the authority needs to give always better and better condition to keep the peripheral country away from default. As a consequence, the tax burden for investors rises and eventually it is such that it perfectly counteracts the benefit of the intervention. At that point the intervention stops, and the peripheral country defaults.

Next, we characterize the ex-ante equilibrium. In fact, when the possibility of the intervention is known, the bond price will reflect the benefit/cost of the future possible interventions. Specifically, we show that the ex-ante bond price is generally higher than absent intervention, since the value of the bond carries the information that claims are insured by the policy intervention. As a consequence, bond prices increase and the actual intervention is delayed since the peripheral country is now able to borrow from the market at a relative low cost. This prediction is in line with the empirical evidence that highlights the effects of the ECB announced programs on reducing sovereign spreads and the increased debt levels of troubled economies. Interestingly, the size of the appreciation of the bond price crucially (inversely) depends on how strongly the tax burden implemented to finance the intervention targets bond holdings.

The final part of the paper concerns welfare. Is the policy ex-ante welfare improving? The first step to answer this question is realizing that the ex-ante welfare of investors and of the peripheral country are both a function of the size of the bond price appreciation caused by the policy. However, while the country's welfare is an increasing function of the size of the appreciation, investors' welfare is a decreasing function. Intuitively, if the policy causes an ex-ante large appreciation of the bond, the country can enjoy much better market borrowing condition and, therefore, higher consumption. However, in this case the fact that the market is willing to offer good borrowing condition delays the actual policy intervention, which, eventually, occurs when the country level of debt is rather large and the ex-post benefits for investors become small. Then, we show that there exists a set of fiscal policies that, by targeting strongly enough bond holding, make the intervention ex-ante Pareto improving. Recall that relaxing the extent to which the tax to finance the policy targets bond holdings increases the bond price appreciation ex ante. This appreciation creates a transfer from investors' welfare to the peripheral country's welfare. There are sets of fiscal policies that are Pareto-efficients and that are linked to financing the intervention by taxing somewhat aggressively bondholders; in contrast, when attenuating too much the dependence of taxes to bondholding, the intervention leaves investors worse off. For example, a policy that is financed via lump-sum taxes is never Pareto improving.

The final comment concerns the essence of the policy intervention that addresses the *non-exclusivity externality*, which is fundamentally a transfer from existing creditors (who pay the taxes) to new investors that underwrite bonds on the primary market (who enjoy the subsidy). Whereas a subsidy on investment is a policy that is not controversial as it can be easily implemented using market instruments, the ability of the policymaker to tax bondholding could be disputed. Nevertheless, once the mechanism is clear, it is then easy to think to alternative arrangements that can lead to the same outcome. Similarly to our policy, we can think that a credit line is provided directly by an international institution that finances the intervention through taxes. Alternatively, we could think to more nuanced contractual arrangements. For example, we can think to a complex seniority structure that, after some debt level, gives progressively increasing seniority to new bondholders, or equivalently that debt is progressively restructured conditional on the country continuing to borrow from the market.

1.1.1 Related Literature

We have already mentioned that a contribution of our paper is to provide an additional rationale for policy interventions in the sovereign bond market, in addition to the equilibrium selection motive as in Gertler and Kiyotaki (2015), Lorenzoni and Werning (2014), Broner et al. (2014), Corsetti, Dedola, et al. (2016), De Grauwe and Ji (2013), and Diamond and Dybvig (1983), and the debt dilution problem as in Bolton and Jeanne (2009), Chatterjee and Eyigungor (2012), Hatchondo, Martinez, and Sosa-Padilla (2016). The *non-exclusivity externality* we introduce in this paper is in the same spirit of Hellwig (1977), although in his framework debt limits are exogenous whereas in our setting they are endogenously determined by the optimal default decision.

The *non-exclusivity externality* can coexist with, but is independent from, the debt-dilution externality. Interestingly, the solution for solving the debt-dilution externality is rather different to the one that mitigates the externality in our paper. In fact, Hatchondo, Martinez, and Sosa-Padilla (2016) show that a way to eliminate the debt-dilution problem features a tax on debt for the bond-issuing country that benefit long-term holders. In our framework, instead, we show that investors themselves are willing to finance a policy that eliminates the market failure, as it will become clear later.

Our paper can be related also to Hatchondo, Martinez, and Padilla (2014), which shows that a sovereign model a la Eaton and Gersovitz (1981) can account for what they label “voluntary debt exchange", i.e. episodes in which both the government and its creditors are likely to benefit from reductions in the government's debt burden. In their setting the debt exchange is ex-post beneficial, but it can be ex-ante detrimental. Although the welfare implications of their policy is similar to the one found in this paper, the reasons underlying the results are very different. In fact, our model can be interpreted, instead, as a “voluntary interest rate reduction", which cannot be implemented by the market but that can be only implemented by a policy that solves a pecuniary externality proper of a competitive market in which the ownership of debt is dispersed. We also show that absent initiatives to coordinate creditors, the government defaults at a sub-optimally low level of debt and policies aimed at solving this externality may largely reduce spreads and extend borrowing limits.

It is also important to highlight what our paper and our results are silent about. Our welfare analysis considers only the direct effects of solving the *non-exclusivity externality*, while we ignore other indirect channels that possibly affect the overall welfare of the economy. For example, lower sovereign spreads may have a beneficial effect on the real economy, through the link between sovereign bonds and the balance-sheet of the banking sector, as highlighted in Gennaioli, Martin, and Rossi (2014), and in Popov and Van Horen (2015), among others.

The rest of the paper is structured as follows. In section 1.2 we introduce a simplified two period model to highlight the mechanism driving the externality. In section 1.3 we outline the general environment in continuous time. In section 1.4 we describe the competitive equilibrium. In section 1.5 we describe the policy intervention and its ex-post properties. In section 1.6 we discuss the ex-ante equilibrium properties. In section 1.7 we characterize the ex-ante welfare implications of the policy. In section 1.8 we conclude with final remarks.

1.2 Two-period Model

The economy lasts two periods: $t = 1, 2$. A representative agent (henceforth government) in a peripheral country issues non-contingent bonds to smooth her consumption. In period 1 the economy has low endowment, $y_1 = y_L$. In period 2 the endowment could be either high, $y_2 = y_H$, or low, $y_2 = y_L$, respectively with probability p and $1 - p$. The country starts with a level of asset $B_1 < 0$, which means that the country has some initial debt. The government utility function, denoted with $u(\cdot)$, is increasing, concave, and satisfies the standard Inada conditions. We assume a logarithm utility function $u(c) = \log(c)$, where c denotes consumption. The concavity of the utility function, together with the assumption that the period-2 income might be higher than the period-1 income, provide borrowing incentives for the government. The government can default on its debt in period 1 or in period 2 provided that endowment is low. However, we assume that the government cannot default if income is high. This assumption aims to capture in a reduced form the fact that it might be too costly for the government to default in a boom, so that a high income realization effectively acts as a commitment technology not to default. We indicate with $\mathbb{1}_t^D$ the default decision at time t, where $\mathbb{1}_t^D = 1$ denotes default and $\mathbb{1}_t^D = 0$ denotes repayment. Default implies no penalty other than exclusion from financial markets.

As standard, we assume that atomistic foreign creditors have access to an international credit market in which they can borrow or lend as much as needed at a constant international interest rate, which we assume to be zero. They have perfect information regarding the economy's endowment process and can observe the level of income every period. Creditors are assumed to price defaultable bonds in a risk neutral manner such that in every bond contract offered they break even in expected value.

The problem for the government is:

$$\max_{\{c_1,c_2,B_2,\mathbb{1}_1^D,\mathbb{1}_2^D\}} \log(c_1) + \mathbb{E}\log(c_2) \tag{1.1}$$

$$s.t. \quad c_1 + qB_2 = y_L + (1 - \mathbb{1}_1^D)B_1, \tag{1.2}$$

$$B_2 = 0 \quad \text{if} \quad \mathbb{1}_1^D = 1, \tag{1.3}$$

$$c_2 = \begin{cases} y_H + B_2, & prob = p \\ y_L + (1 - \mathbb{1}_2^D)B_2, & prob = 1 - p. \end{cases} \tag{1.4}$$

Some remarks are in order. First, without loss of generality we have assumed that there is no discounting from period 1 to period 2. Second, q denotes the bond price. Third, defaulting in period 1, i.e. $\mathbb{1}_1^D = 1$ implies that the government does not repay its initial debt, B_1, and that it is excluded from the financial market so that in that case necessarily $B_2 = 0$.

1.2.1 Competitive Equilibrium

Definition 1. A Competitive Equilibrium for this economy is defined as a set of policies for consumption in period 1, c_1, and in period 2, c_2, for government's asset holdings B_2, a default decision in period 1 and period 2, $\mathbb{1}_1^D$ and $\mathbb{1}_2^D$, and a bond price q, such that:

1. Taking as given the bond price q, the government's asset holdings and default decisions satisfy the government optimization problem.

2. The bond price, q, being consistent with creditors' expected zero profits, reflects the government's period-2 default probability.

3. Taking as given the government policies, consumption satisfies the resource constraint.

To characterize the competitive equilibrium, we solve the model by backward induction. Notice that defaulting in period 2, i.e. $\mathbb{1}_2^D = 1$, simply implies that the government will not repay its debt and no further penalties occur. As a result, the government will default in the low income state every time $B_2 < 0$. Formally, we can state:

Lemma 1. *If $B_2 < 0$ and $y_2 = y_L$, then $\mathbb{1}_2^D = 1$.*

It is easy to show that, given our setup, the consumption smoothing motive of the government implies that if he starts with some legacy debt in period 1, $B_1 < 0$, it will never choose to run a budget surplus. This is true because, even though the bond price is so low that the government does not find convenient to borrow further, $B_2 \geq B_1$ is strictly dominated by defaulting and implicitely set $B_2 = 0$. Hence the following must be true:

Lemma 2. *If $B_1 < 0$, then $B_2 \leq 0$.*

Equipped with these two results, we can characterize the optimal default decision in period 1. The value of not defaulting in period 1 is

$$V_1^{ND}(B_1, q) = \max_{B_2} \log(y_L + B_1 - qB_2) + (1-p)\log(y_L) + p\log(y_H + B_2). \tag{1.5}$$

Notice that we have used the fact that Lemma 1 together with Lemma 2 imply $\mathbb{1}_2^D = 1$. Solving for B_2, gives the optimal asset/debt position:

$$B_2^*(B_1, p) = \frac{\frac{p}{q}(y_L + B_1) - y_H}{1+p}. \tag{1.6}$$

If instead the government defaults in period 1, its value, V_1^D is:

$$V_1^D = \log(y_L) + (1-p)\log(y_L) + p\log(y_H). \tag{1.7}$$

In a competitive market the bond price is equal to the period-2 probability of repayment, which is the probability to obtain a high income realization; therefore, $q = p$. Hence, the government will optimally default wheneven $V_1^D \geq V_1^{ND}(B_1, p)$. We are now ready to prove the main proposition that characterizes the competitive equilibrium: there exist a threshold $\bar{B}_1$ such that if B_1 is above that threshold the government does not default in period 1, while if B_1 is below the threshold, the government defaults in period 1.

Proposition 1. *In a competitive equilibrium* $\exists!\bar{B}_1 < 0$ *such that:* $B_1 \leq \bar{B}_1 \iff \mathbb{1}_1(p, B_1) = 1$.

Proof. In the competitive equilibrium $q = p$ and the period-1 default condition is: $V_1^D \geq V_1^{ND}(p, B_1)$. The optimal asset decision, evaluated at $q = p$ is: $B_2^*(p, B_1) = \frac{(y_L+B_1)-y_H}{1+p}$. Substituting into the non-default value in period 1 and doing some simple algebra, the government decides to default if and only if:

$$\frac{\log(y_L) + p\log(y_H)}{1+p} \geq \log\left(\frac{y_L + py_H + B_1}{1+p}\right). \tag{1.8}$$

Assume B_1=0, by Jensen's inequality, the concavity of the logarithm function implies that equation (1.8) is not satisfied and therefore a necessary condition for the government to default is that $B_1 < 0$. Since the RHS of (1.8) is monotonically increasing and continuous in B_1, there exists a unique threshold

$$\bar{B}_1 = (1+p)\left(y_L y_H^p\right)^{\frac{1}{1+p}} - (y_L + py_H) < 0$$

such that the government defaults if and only if $B_1 \leq \bar{B}_1$.

□

1.2.2 Non-Exclusivity Externality

We now show that the competitive market is characterized by an externality. Assume that the initial level of debt is $\bar{B}_1$, and the government, being at the indifference point between defaulting and not defaulting, will default in period 1. In this case, existing lenders are going to lose their investment. Indeed, since debt is non-exclusive and lenders are atomistic, they cannot affect the competitive price $q = p$. Notice, in fact, that the reason why the country will default in period-1 is because the country can only borrow at a price $q = p$. Everything else equal, better borrowing conditions would avoid default. In this section we show: (i) that agents in the economy would all be better off if the bond price was slightly higher than the market price; and (ii) how to simply implement that price.

First, we can show that the value of non-defaulting in equation (1.5) is a positive function of the bond price q.

Lemma 3. $B_1 < 0 \Rightarrow \frac{\partial V_1^{ND}(B_1,q)}{\partial q} > 0.$

Proof. By the envelope theorem, taking the derivative of (5) with respect to the bond price, we get

$$\frac{\partial V_1^{ND}(B_1,q)}{\partial q} = -\frac{B_2^*(q,B_1)}{y_L + B_1 - qB_2^*(q,B_1)}$$

The denominator $y_L + B_1 - qB_2^*(q,B_1)$ from the government budget constraint is equal to c_1 which must be greater than zero because of the Inada conditions. Then $B_2^*(q,B_1) < 0$ implies that the value of non-defaulting is increasing in the bond price. $\square$

Since by definition of $\bar{B}_1$, $V_1^{ND}(\bar{B}_1,p) = V_1^D$, then $V_1^{ND}(\bar{B}_1,p+\delta) > V_1^D$. Therefore, if the country faces the price $p+\delta$, $\forall\delta > 0$ the government would not default, it would stay in the market and optimally borrow the quantity $\tilde{B}_2 = \frac{\frac{p}{p+\delta}(y_L+\bar{B}_1)-y_H}{1+p}$. [3]

[3]Notice that $\tilde{B}_2 < 0$ only when $\delta < \frac{p(y_H-y_L-B_1)}{y_H}$, which puts an upper bound on δ.

Country's Welfare. Obviously, the country is better off under the new price. Let's define as $W^c(p+\delta) - W^c(p)$ the government's welfare differential. Since $V_1^{ND}(\bar{B}_1, p+\delta) > V_1^{ND}(\bar{B}_1, p) = V_1^D$, then:

$$W^c(p+\delta) - W^c(p) > 0.$$

Old Investors' Welfare. Now let's look at existing period-1 creditors. Under competitive price $q = p$, they will lose all their investment, and therefore their payoff is equal to zero. Under the alternative price $p + \delta$, they will get back their investment $-B_1 > 0$. Given that they are risk neutral, their welfare gain from the higher price is:

$$W^{oldI}(p+\delta) - W^{oldI}(p) = -B_1 > 0.$$

New Investors' Welfare. Now let's look at new investors. First recall that the economy will default surely if in period-2 the realization of income is low. Hence, if new investors buy the bond price at $\tilde{q} = p + \delta$, they would make an ex-ante loss, equal to

$$W^{newI}(p+\delta) - W^{newI}(p) = -\delta(-\tilde{B}_2).$$

The competitive market, per-se, cannot support a price higher than p. However, it can be implemented by introducing a simple subsidy to acquire new issuance of bonds, which can be financed by taxing old investors. Recall, that old investors enjoy a rather large welfare gain if the equilibrium market price of the bond is higher than p. Indeed, the fact that default ivs a binary choice generates a discontinuity in the payoff to old investors.

Assume that old investors subsidize the purchase of new bond issuance and the subsidy for unit of bond is equal to δ. We will show that under this transfer the market can sustain a Pareto improving outcome.

Proposition 2. *The competitive equilibrium is suboptimal. There exists $\bar{\delta}$ such that for $\delta \in (0, \bar{\delta}]$ a Pareto improvement over the competitive equilibrium can be obtained.*

Proof. The equilibrium price of the bond under the subsidy is: $\bar{q} = p + \delta$. Investors internalize the subsidy and break even in expectation. The welfare of the country is increasing in δ since the new price relaxes the government budget constraint. Are existing investors now willing to finance the subsidy? With a strictly positive subsidy δ, the country will not default. Their welfare gain from introducing the subsidy is:

$$W^{oldI}(p+\delta) - W^{oldI}(p) = -B_1 \mathbb{1}_{\delta>0} - \delta(-\tilde{B}_2)$$

The first term is the revenue in period 1 that occurs only when δ is strictly positive, since only in that case lenders will get back their original investment; the second term is the cost of the transfer, which is equal to the unit cost of the subsidy, δ, and the total amount of new bond optimally sold by the country and acquired by new investors, equal to $-\tilde{B}_2$. Using (2) and taking the limit as $\delta \to 0$, we have that

$$\lim_{\delta \to 0} \left[W^{oldI}(p+\delta) - W^{oldI}(p) \right] = -B_1$$

This discontinuity and the fact that the welfare gain is continuous in δ prove the result

□

Figure 1.1 shows the welfare of existing investors as a function of δ. At the competitive price, $q = p$, and therefore $\delta = 0$, existing investors make a loss since the country defaults. A price higher than p creates a jump on welfare since the country will not default and investors will be repaid. Then, the higher is the unit subsidy, the higher is the cost for investors, since not only the unit subsidy obviously increases, but also the amount of issued bond B_2 increases, since the country will optimally demand more debt when borrowing conditions improve. Importantly, our result shows that small deviations from the competitive price $q = p$ are Pareto efficient since both the country and old investors are better off, while new investors are indifferent.

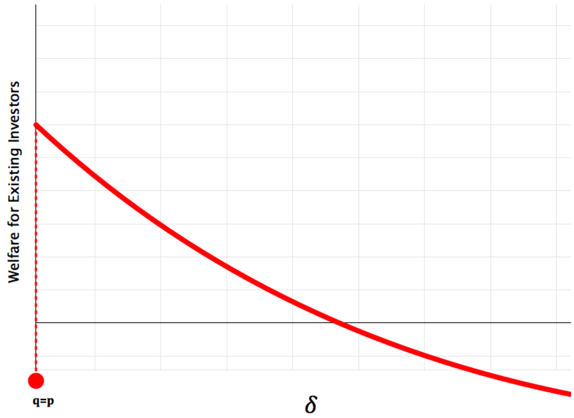


FIGURE 1.1: Existing investors' welfare as a function of the subsidy δ

This two-period model shows the heart of the non-exclusivity externality: the competitive market cannot price the incentives of old investors to avoid default and that results in a loss in efficiency. However, this simple model cannot answer interesting questions related to the proposed solution for the externality, such as whether the policy is effective over longer horizons, whether default will be always avoided under such a policy, and what are the ex-ante effects of the policy once investors are aware of it. In order to answer these questions we now introduce a more complete infinite horizon model in continuous time.

1.3 Continuous time model

The Peripheral Economy and Uncertainty. A representative agent (henceforth government) in a peripheral country issues non-contingent bonds to smooth her consumption. The consumption smoothing desire is motivated by the uncertainty about the exogenous income that the government is facing and that is the only source of uncertainty in the economy. We assume that time is continuous and the income process Y_t follows a continuous-time Markov chain, as conventionally defined, i.e.:

Definition 2. Continuous-time Markov chain. A continuous-time Markov chain with finite or countable state space $\mathcal{Y}$ is a family $\{Y_t = Y(t)\}_{t\geq 0}$ of $\mathcal{Y}$-valued random variables such that:

(a) The paths $t \mapsto Y(t)$ are right-continuous step functions; and

(b) For all $t \geq 0, s \geq 0, i \in \mathcal{Y}, j \in \mathcal{Y}$,

$$P\left(Y(s+t) = j | Y(s) = i, \{Y(u) : 0 \leq u < s\}\right) = P\left(Y(s+t) = j | Y(s) = i\right).$$

Condition (a) guarantees that the Markov chain makes only finitely many jumps in any finite time interval. Condition (b) is the natural continuous-time analogue of the Markov property. It requires that the future is conditionally independent of the past given the present.

More specifically, we assume a two-state process, i.e. $\mathcal{Y} = \{y_L, y_H\}$: here y_L denotes a bad state in which income is low and $y_H > y_L$ denotes a good state in which income y_H is high. In the initial period, $t = 0$, the economy is in the bad state (recession): the government is poor and needs to borrow to finance consumption and satisfy coupon payments. Eventually the country recovers and jumps to the good state. However, the time in which the country exits the recession is uncertain. Once the country recovers, uncertainty is resolved and the country will remain in the good state forever after. These assumptions impose restrictions on the *infinitesimal generator* matrix that governs the transition of the process. It can be shown that the transition probability matrix in this case is:

$$P(t) = \begin{pmatrix} e^{-\lambda t} & 1 - e^{-\lambda t} \\ 0 & 1 \end{pmatrix},$$

with initial condition $y(0) = y_L$. The key remark is that the time of the jump from the low to high income, which we define as T^j, has exponential distribution with parameter λ.

We have, therefore, a two-stage game. In stage-1, the prospect of an income increase provides a motive for borrowing. Uncertainty is then fully resolved at some random date at which point we enter stage-2 and the government receives a constant stream of income y_H.

Remark 1. *This setting, which is consistent with Hellwig (1977), allows us to derive analytic results. Assuming an absorbing high income state is not a key limitation, since we are, anyway, mainly interested in the dynamics during the low income state, which are obviously the main drivers of sovereign crisis and monetary intervention.*

Asset Structure. The government issues non-contingent bonds. These bonds have coupons that decrease at a continuum rate δ. Hence, a bond issued at t promises to pay the sequence of coupons:

$$ke^{-\delta(s-t)}, \quad \forall s \geq t,$$

where $\delta \in (0, 1)$ and $k > 0$. We normalize and set $k = \delta + r$, so that the bond price is equal 1 when the risk of default is zero at all future dates, and where r is the assumed risk-free rate in the economy. This well-known formulation of long term bonds is useful because it avoids having to carry the entire distribution of bonds of different maturities (see Hatchondo and Martinez (2009)). A bond issued at $t - j$ is equivalent to $e^{-\delta(t-j)}$ bonds issued at t, so the vector of outstanding bonds can be summarized by a single state variable b_t, which is equal to total debt in terms of equivalent newly issued bonds. The parameter δ controls the maturity of debt, with $\delta = 1$ corresponding to the case of short-term bond and $\delta = 0$ corresponding to the case of a consol.

Government and Default. We allow for the government to endogenously default on its debt obligation. A key simplifying assumption for our analysis is that default can occur only when income is in the low state. Hence, by assumption we rule out default when the economy exits the recession phase. As it will be clear throughout the paper, we will focus our analysis mainly in the recession state, since it is arguably the time in which policy intervention is meaningful; hence, we believe that simplifying the dynamics of the model in the high income state does not bear a large cost.[4] We assume the following sequence of events: if the government defaults, which always happens when income is still in the low state y_L, the government stops any coupon repayment and the country is excluded from the financial market so that the economy lives in autarky. When the recession is over, which means when income jumps to the higher state y_H, the government renegotiates debt payments by repaying only a fraction $\phi \in [0, 1]$ of outstanding debt, and it gains back access to financial markets.

We denote with T the default period. In the next section we characterize the choice of the optimal time of default; here we describe the constraints the government faces. There are two cases, then: (i) either the country jumps out of the recession at a time, T^j, which occurs after the time of default, T and the country defaults on its debt; (ii) or default never happens. In the first case,

[4]This assumption, which allows us to derive analytical results, is in line with the vast empirical literature on sovereign defaults that links default episodes to periods of recession. Using quarterly data for 39 developing countries over the 1970-2005 period, Yeyati and Panizza (2011) show that defaults are associated with deep recessions; Tomz and Wright (2007) analyse defaults in a longer sample,1820-2004, and, although they find evidence that defaults also happens without severe recessions, the maximum default frequency occurs when output is at least 7 percent below trend. Hence, we believe that assuming no defaults in the good state of the economy is quite realistic. One can relax this assumption by assuming an output cost of default and a risk of returning to the low state of the economy after the jump to the high state. This setting would be more similar to standard business cycle endogenous default models as in Arellano (2008), which requires numerical solutions.

$T^j > T$: the government budget constraint is:

$$
\begin{aligned}
c(t) + q(t)\left[\dot{b}(t) + \delta b(t)\right] &= y_L + (r+\delta)b(t), && \text{for } t < T \text{ and for a given } b(0),\\
c(t) &= y_L, && \text{for } T \leq t < T^j,\\
c(t) + q(t)\left[\dot{b}(t) + \delta b(t)\right] &= y_H + (r+\delta)b(t), && \text{for } t \geq T^j \text{ and with } b(T^j) = \phi b(T).
\end{aligned}
$$

The first equation states the resource constraint prior to the default. $c(t)$ denotes consumption at time t, $b(t)$ denotes asset holding, $\dot{b}(t)$ denotes the instantaneous change in asset position, $q(t)$ is the bond price. The second equation indicates that the government is excluded from the financial market from the time of default, T, to the time in which it enters in the good economic state, T^j. The third equation describes the budget constraint from the time of the jump onwards. Two things are worth noticing; first, when the economy regain access to the financial market it starts with a renegotiated level of debt, $b(T^j) = \phi b(T)$;[5] second, since by assumption after the jump no default will occur, then $q(t) = 1, \forall t \geq T^j$, because we have normalized the price of a risk-free bond to unity.

In the second case, $T^j < T$, there is no default, and the government budget constraint is:

$$
\begin{aligned}
c(t) + q(t)\left[\dot{b}(t) + \delta b(t)\right] &= y_L + (r+\delta)b(t), && \text{for } t < T^j \text{ and for a given } b(0),\\
c(t) + q(t)\left[\dot{b}(t) + \delta b(t)\right] &= y_H + (r+\delta)b(t), && \text{for } t \geq T^j.
\end{aligned}
$$

Investors. The economy is populated by a continuum of mass 1 of risk neutral atomistic and homogenous investors, which operate in a competitive financial market, and, therefore, take the bond price as given. Let $a(t)$ denote each investor's individual bond holdings, which, in our economy, are the counterpart of governments' bond, so that in equilibrium we will have that $a(t) = -b(t)$. Denote with $V(\cdot)$ the investor's value of asset holding. Then, the investor's problem at any time t before the jump and before default, i.e.

[5] Recall that in our notation $b(t)$ denotes asset level, so that at debt is negative asset holding.

$\forall t < \min\{T, T^j\}$, is:

$$V\left(a(t)|\{q(s)\}_{s=t}^{T}\right) = \max_{\{a(s)\}_{s=t}^{T}} \int_t^T \left(-q(t)(\dot{a}(s) + \delta a(s)) + (r+\delta+\lambda)a(s)\right) e^{-(r+\lambda)(s-t)} ds + \\ + V\left(a(T)|q(T)\right) e^{-(r+\lambda)(T-t)}. \tag{1.9}$$

We assume that investors cannot short sell the bond and face the following nonegativity constraint

$$a(t) \geq 0,\ \forall t.$$

The integral captures the value of a bond throughout the uncertain times in which the economy is in a recession and the government might default at time T; in this time interval, the investor can increase her asset holding position at the price $q(s)$, with $t \leq s \leq T$, and this investment returns the coupon repayment, $r + \delta$, as well some capital gain in case the economy jumps in the higher income state, event with arrival rate λ. Notice, in fact, that the assumption about the income process makes y_H an absorbing state and, therefore, once the income has jumped in that state the government will never default, and, therefore, $q(t) = 1$, $\forall t \geq T^j$. On the contrary, default risk while income is low, implies that $q(t) \leq 1$, $\forall t \leq T$. Finally, the last term captures the value of the bond in case the government defaults, which includes future repayments when the economy exits financial autarky and renegotiates the debt payments.

Our setting leads to some straightforward results:

Proposition 3. *Bond Price. The bond price $q(t)$ satisfies the following conditions:*

1. *For any period before default/jump, that is $\forall t < \min\{T, T^j\}$, the law of motion of the price $q(t)$ satisfies:*

$$\dot{q}(t) = (r + \delta + \lambda)(q(t) - 1),$$

2. *The bond price at default, $q(T)$, is:*

$$q(T) = \frac{\phi\lambda}{r+\lambda}.$$

This price is also the market price in any period between default, T, and the time of the jump to the high income state, T^j.

3. *For any period after the jump, that is $\forall t \geq T^j$, the law of motion of the price $q(t)$ satisfies:*

$$q(t) = 1, \tag{1.10}$$

See Appendix A.1 for the proof. Condition 3 follows directly from the assumption that the government cannot default in the high income state. Hence, after the jump to the high income state occurs, the sovereign bond is equivalent to a risk-free asset. Condition 2 relates the price of the bond at an instant prior to default directly to the recovery rate of the bond, ϕ, and to the probability that the bond will be paid in full, which is linked to the probability λ that the country obtains a good realization of income in that instant. Condition 1 is the non-arbitrage condition derived for risk-neutral investors acting in a competitive market. Conditions 1 and 2 are at the heart of the *non-exclusivity externality*. In a competitive market investors are price takers, they are willing to underwrite a new bond only if the price is lower or equal to the present value of future repayments on that specific bond. A new bond can never be sold at a higher price even though that higher price may increase the value of existing bonds. That is because it is individually rational for each existing bondholder, being atomistic and anonymous, to shun the new issuance trying to free ride on the increase in value of the bonds already in its portfolio. Essentially it is as if bonds are priced by new investors at each point in time. As it will be clear later, this feature, when combined with market incompleteness generates a pecuniary externality that makes the government default at an inefficiently too low level of debt.

1.4 Debt and Price Equilibrium Dynamics

In this section we characterize the equilibrium path of debt, bond price, and default time resulting from the government's optimization problem. As described before, the government starts with low income y_L and with instantaneous probability λ income jumps to the higher level y_H, at which point uncertainty is resolved and the government receives y_H forever after.

The value at the jump. Let us first derive the value of the government after uncertainty is resolved, i.e. when income jumps to the absorbing high state. In order to obtain analytical results, we assume that the instantaneous utility is $u(c) = \log(c)$ and that the risk-free rate in the economy is equal to the discount factor, $r = \rho$. These assumptions imply that after the jump, since there is no uncertainty, the government will optimally maintain a constant consumption.

If the government has not defaulted prior to the jump, the problem is:

$$W^j\left(b(T^j)\right) = \max_{\{c(t)\}_{t\geq T^j}} \int_{T^j}^{\infty} e^{-\rho(t-T^j)} \log(c(t))dt$$
$$\text{s.t. } \dot{b}(t) = y_H - c(t) + (\rho+\delta)b(t) - \delta b(t),$$

where we have used the fact that after the jump $q(t) = 1, \forall t \geq T^j$. The solution of this trivial problem gives the value at the moment of the jump, that is:

$$W^j\left(b(T^j)\right) = \frac{\log(y_H + \rho b(T^j))}{\rho}.$$

If the government has already defaulted prior to the jump, the problem is identical beside the fact that at the moment of the jump the country reenters the financial market with a level of assets that it is a fraction ϕ of its obligation at the moment of default, $b(T)$. Hence, it will start the period of the jump T^j with a level of assets equal to $\phi b(T)$.

Therefore, defining with x the starting level of assets at the time of jump T^j, we can conveniently write the value of the government at T^j as:

$$W^j(x) = \frac{\log(y_H + \rho x)}{\rho} \quad \text{with:} \begin{cases} x = b(T^j) & \text{if } T^j \leq T, \\ x = \phi b(T) & \text{if } T^j > T. \end{cases} \tag{1.11}$$

The value at default. If the government defaults at time T, then it will remain in autarky consuming the low level of income until the period of the jump, at which point it enjoys the value $W^j(\phi b(T))$ as measured above. Hence, the value function at default as a function of the level of asset $b(T)$ is:

$$W^d(b(T)) = \frac{\log(y_L) + \lambda W^j(\phi b(T))}{\rho + \lambda} \tag{1.12}$$

Government problem prior to default. We are now ready to write the problem of the government, its value, and its default decision, when it faces a low income and has not yet defaulted. The government takes the path of the bond price as given and it chooses optimally the path for consumption $\{c(s)\}_{s=t}^{T}$ and the optimal time of default T, as follows:

$$W(b(t)) = \max_{\{c(t)\}_{s=t}^{T}, T} \int_t^T e^{-(\rho+\lambda)(s-t)} \left[\log(c(s)) + \lambda W^j(b(s))\right] ds + \tag{1.13}$$

$$+ W^d(b(T)) e^{-(\rho+\lambda)(T-t)}, \tag{1.14}$$

$$\text{s.t. } \dot{b}(t) = \frac{1}{q(t)} \left(y_L - c(t) + (\rho + \delta) b(t)\right) - \delta b(t), \tag{1.15}$$

$$\dot{q}(t) = (\rho + \delta + \lambda)(q(t) - 1), \tag{1.16}$$

$$q(T) = \frac{\lambda \phi}{\lambda + \rho}, \tag{1.17}$$

$$b(t) \text{ given.}$$

The first constraint is the resource constraint of the government. The second constraint is the evolution of the bond price that follows from the investors' problem. The third constraint is the equilibrium bond price at time of default.

The continuous time Euler equation that characterize the equilibrium is:

$$\frac{\dot{c}(t)}{c(t)} = \frac{\lambda}{q(t)} \left[c(t) W_b^j(b(t)) - 1 \right], \quad \forall t \leq T, \tag{1.18}$$

where $W_b^j(\cdot)$ denotes the derivative of the function $W^j(\cdot)$ with respect to b. See Appendix A.2 for the formal derivation.

Terminal conditions. The dynamic differential equation in (1.18), together with the differential equation for $\dot{b}(t)$ coming from the government resource constraint in (1.15) and the evolution of the bond market price for $\dot{q}(t)$ in (1.16), pins down the optimal path of consumption and asset holding, given the terminal conditions for the three variables. Deriving these terminal conditions in the context of *free terminal time boundary value problems* is well established. A formal derivation is provided in Hartl and Sethi (1983) and applied in Hellwig (1977) in a similar context.

In our case, the system of the three terminal conditions, for $b(T)$, $c(T)$, and $\dot{b}(T)$ is:

$$\log(c(T)) - \log(y_L) = \lambda \left[W^j(\phi b(T)) - W^j(b(T)) \right] - W_b^d(b(T))\dot{b}(T) \tag{1.19}$$

$$c(T) = y_H + \rho\phi b(T), \tag{1.20}$$

$$\dot{b}(T) = \frac{\rho + \lambda}{\lambda\phi} \left[y_L - c(T) + (\rho + \delta) b(T) \right] - \delta b(T). \tag{1.21}$$

See Appendix A.3 for the formal derivation

The first equation should be interpreted as a trade-off in the time dimension and pins down the default time. The left hand side represents the benefit of delaying default of one instant, which stems from the possibility to consume more than in autarky. The right hand side represents the cost of delaying default of one instant, which is composed by two terms: (i) the foregone opportunity to default in case the jump occurs at that instant, which is a function of the arrival rate, λ, and of the renegotiation parameter ϕ; and (ii) the disutility to increase the debt burden. The second equation pins down the quantity of consumption and debt at default. The solution of this system of three equations in three unknown, $b(T)$, $c(T)$, $\dot{b}(T)$, determines these terminal values.

Competitive Equilibrium. We are now ready to define a competitive equilibrium for the economy, prior to the default or jump.

Definition 3. A Competitive equilibrium is a bond price sequence $\{q(t)\}_{t=0}^{T}$, a saving sequence $\{b(t)\}_{t=0}^{T}$, a consumption sequence $\{c(t)\}_{t=0}^{T}$, and an optimal default time T for the peripheral government, and an asset holding sequence $\{a(t)\}_{t=0}^{T}$ for investors, such that, given $\{q(t)\}_{t=0}^{T}$:

(i) investors solve the problem in (1.9).

(i) the government solves problem in (1.14)-(1.17).

(i) the government defaults at T, if $T < T^j$.

(i) bond markets clear, i.e. $b(t) = -a(t), \forall t$.

Once again, for convenience, we focus on the equilibrium for any $t < \min\{T, T^j\}$, since this is the relevant case for which the policy intervention is meaningful. It is trivial to define and derive the equilibrium condition in those cases: the bond price after default or after the jump are described in Proposition 3, whereas the government budget constraints are described in Section 1.3. Since these cases are not relevant for the scope of the paper we omit their formal description.

Hence, the equilibrium before default is characterized by the following system of differential equations:

$$\begin{aligned}
\dot{q}(t) &= (\rho+\delta+\lambda)(q(t)-1), \quad \forall t \leq T \\
q(t)\dot{b}(t) &= y_L - c + b(t)[\rho + \delta(1-q(t))], \quad \forall t \leq T \\
\frac{\dot{c}(t)}{c(t)} &= \frac{\lambda}{q(t)}\left[c(t)W_b^j(b(t)) - 1\right], \quad \forall t \leq T \\
q(T) &= \frac{\lambda\phi}{\rho+\lambda}, \\
c(T) &= y_L + \rho\phi b(T), \\
\log(c(T)) - \log(y_L) &= \lambda\left[W^j(\phi b(T)) - W^j(b(T))\right] - W_b^d(b(T))\dot{b}(T), \\
& b(0) \text{ given},
\end{aligned}$$

where $W^j(\cdot)$ and $W^d(\cdot)$ are defined respectively in equation (1.11) and (1.12).

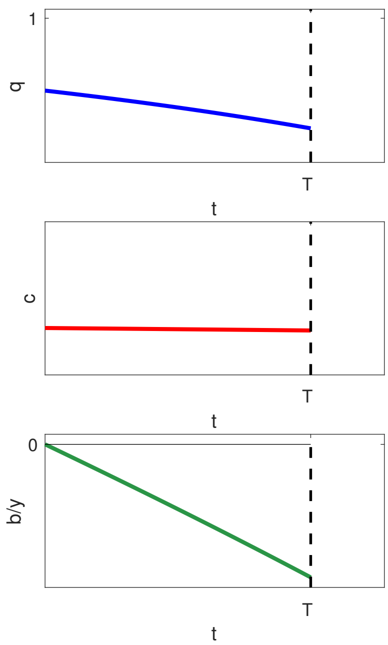
The competitive equilibrium is then obtained as a solution of a well-known problem in physics and engineering, called boundary value problem. Intuitively, given the solution for the terminal conditions at T, the solution of the system finds a path for the $\dot{b}(t)$, $c(t)$, $q(t)$, and therefore for $b(t)$, that links the terminal conditions to the given initial value $b(0)$ through the equilibrium path.[6]

In Figure 1.2 we plot the equilibrium path of the bond price $q(t)$ and of the government's level of asset to output, $\frac{b(t)}{y_L}$, and consumption, $c(t)$. To visualize how the economy can reach the default stage, we assume that the recession is long lasting and income does not jump to the high state before default. During the recession, in order to keep a roughly steady level of consumption, the government must issue bonds. While debt increases, default incentive rises and investors continuously devalue the bond. In turns, a lower bond price requires a larger amount of debt to finance consumption. This vicious circle

[6] A numerical solution for the boundary value problem can be computed in matlab using the function bvp4c.m. As standard for non-linear system, it is not trivial to prove the existence and the uniqueness of the solution. Nevertheless, for any calibration of the model we have tried, we were able to always find a unique numerical solution.

continues until the bond price reach the level $q(T) = \frac{\phi\lambda}{\rho+\lambda}$, at which points the government defaults.

FIGURE 1.2: Bond Price and Debt before Default



Note: this graph plots the competitive equilibrium path of the bond price (top-left panel), the level of asset as a fraction of output (bottom-left panel), peripheral economy's consumption (top-right panel) and function ICC (bottom-right panel), as a function of time (x-axis).

1.5 Policy intervention and Ex-Post Equilibrium

In this section, we show that a balanced budget policy intervention, paid by investors, can improve the market outcome. We first focus on the equilibrium ex-post and show that at default, existing creditors would be better off by extending credit to the country at a better price than the market price in order to delay the time of default. We propose a simple and tractable policy that incentivizes new investors to do so. We will then discuss which alternative policies could reach the same goal. In the next section we extend the analysis to the case ex-ante and show how the intervention has a significant impact on bond prices and default thresholds.

Policy The policy we consider is a subsidy on lending financed by taxes on investors. A policy maker sets a subsidy $g(b)$ per unit of bonds underwritten on the primary market. Since the subsidy is internalized by competitive investors, in equilibrium the borrowing government will be offered a price equal to:

$$q^p(b) = q(b) + g(b) \tag{1.22}$$

where q^p is the bond price on the primary market and q is the bond price on the secondary market that in equilibrium will depend both on the subsidy and on the type of tax used to finance the policy. Notice that, for convenience, our notation now implies the current level of asset holding (debt), b, as a state variable. Since there is a monotone relationship between asset holding and time, one can map a level of asset holding b with the time t at which that level is reached. We denote the time at which the policy starts with T^P and from now on we express the problem in recursive form.

Remark 2. *It is important to understand, as it will be clarified later, that by changing $g(b)$ and the tax rule, the policymaker is able to implement any $q^P(b)$ she likes. Therefore, with a slight abuse of notation, in order to simplify the exposition, we will sometime refer to $q^P(b)$ as a policy instrument.*

Let $G(b)$ be the gross subsidy, which is also the total cost of the policy, i.e.:

$$G(b) = -g(b)(\dot{b} + \delta b), \tag{1.23}$$

which simply states that the total cost of the policy is the product of the per-unit loss and the quantity of new bond issuance. We assume that the intervention is financed with a balanced budget by levying taxes on all investors:

$$G(b) = \int_0^1 \tau(i, b) di, \tag{1.24}$$

where $\tau(i, b)$ is a generic function that summarizes the tax rule applied to an investor i.

How does the policymaker set the subsidy? We postulate a subsidy $g(b) \geq 0$ that makes the borrowing government indifferent to default or keep borrowing. This means that, by construction, the policy makes the borrowing government ex post as well off. We then show in the next section that, once accompanied with an optimal stopping time, this policy actually makes creditors always better off ex-post and is, therefore, Pareto improving. Formally, we consider a policy-maker that sets q^P at each point in time in order to make an optimizing government indifferent between continuation and default. The policy can be determined by the solution to the following problem:

$$\begin{aligned} (r+\lambda)W(b) = \; & \max_{c} \; \log(c) + \lambda W^j(b) + W_b(b)\dot{b}, \\ s.t. \quad & q^p \dot{b} = y_L - c + b[\rho + \delta(1 - q^p)], \\ & W(b) = W^d(b). \end{aligned} \tag{1.25}$$

Taking first order conditions, we obtain a system of three equations in three unknowns $(c, \dot{b}, q^P)$:

$$\log(c) - \log(y_L) = \lambda(W^j(\phi b) - W^j(b)) - W_b^d \dot{b}, \tag{1.26}$$

$$\frac{q^P}{c} = W_b^d(b), \qquad , \tag{1.27}$$

$$q^P \dot{b} = y_L - c + b[\rho + \delta(1 - q^P)]. \tag{1.28}$$

Given b, the system pins down the policy functions $c(b), \dot{b}(b), q^P(b)$. The first two equations determine the government indifference condition between borrowing and defaulting, while the last equation is the standard government's budget constraint. Notice that the equations (1.26)-(1.28) are identical, at T, to the terminal conditions (1.19)-(1.21).

The following Proposition characterizes the solution of the dynamic system (1.26)-(1.28).

Proposition 4. *Let us denote with $\dot{b}(b)$ the solution of the saving rate as a function of the level of assets resulting from the system (1.26)-(1.28). And assume a solution of the non-linear system above does exist. If $b(T) < 0$, then there is a unique stable steady state at which the dynamic system (1.26)-(1.28) converges and all the variables remain constant. Moreover, the intervention is characterized by a bond price $q^P(b)$ that is monotonically decreasing in b or, equivalently, monotonically increasing in time.*

See Appendix A.4 for the proof.

The investors' gain and the length of intervention The solution of the dynamic system presented above ignores investors' incentives. Is this type of intervention beneficial for investors? And if so, for how long? On the one hand, bond holders might gain from the government delaying default, but, on the other hand, they have to finance the policy by paying taxes. In this section we quantify the net gain of investors from the policy and, consequently, we pin down the duration of the policy intervention.

The Hamiltonian-Jacobi-Bellman equation of a representative investor who holds the entire stock of debt until maturity and underwrites any new bond issuance is:

$$(r+\lambda)V(-b) = -G(b) + q\left(\dot{b} + \delta b\right) - (r+\delta+\lambda)b - V_{-b}(-b)\dot{b},$$

where we have now incorporated the fact that the cost of the policy, $G(b)$, is a burden for investors. Substituting the expression for $G(b)$ in equation (1.23), and using the budget constraint of the government post intervention in equation (1.28), the expression simplifies to:

$$(r+\lambda)V(-b) = (y_L - c - \lambda b) - V_{-b}(-b)\dot{b}. \tag{1.29}$$

At each point in time, the authority has the option to stop the policy. In that case the government defaults and investors recover

$$V^d(-b) = -q^* b$$

where $q^* = \frac{\phi\lambda}{r+\lambda}$ is the expected recovery value for each bond as defined above. Provided that a debt level b^* and corresponding time T^E does exist at which the authority has incentive to stop the intervention, the terminal conditions related to investors' Hamiltonian-Jacobi-Bellman that need to be satisfied at the optimal stopping time are:

$$\begin{aligned} V(-b^*) &= V^d(-b^*), \\ V_{-b}(-b^*) &= q^*, \end{aligned}$$

The first condition is a *value matching* condition and states that at the margin the value of keep lending to the government should be equal to the value of letting the government default. The second is a *smooth pasting* condition: if value functions do not smooth paste at b^*, then stopping at b^* cannot be optimal. Better to stop an instant earlier or an instant later. Therefore, a policymaker that takes into account investors' utility, and therefore taxpayers' utility, will keep lending to the peripheral country as long as the policy satisfies the following incentive compatibility condition (ICC, henceforth):

$$-(\rho+\lambda)q^* b \le y_L - c - \lambda b - q^* \dot{b}, \tag{1.30}$$

which is obtained by substituting the two conditions above into equation (1.29), and realizing that (1.30) holds with inequality for any $b > b^*$ and with equality for $b = b^*$. The left hand side is the value of letting the country default at the debt level b, while the right hand side is the value of keep lending to the government and let it default one instant later. The condition therefore states that the authority will extend loans as long as the value of delaying default for one instant in time is greater than the value of letting the country default immediately. Simplifying, we define the function $ICC(t)$ as:

$$ICC(t) \equiv y_L - c(t) - \lambda b(t)(1-\phi) - \frac{\phi\lambda}{\rho+\lambda}\dot{b}(t). \tag{1.31}$$

Hence, investors gain from the intervention as long as $ICC(t) > 0$, and the stopping time for the policy is the period T^E such that $ICC(T^E) = 0$.

We can show the following results:

Proposition 5. *The Ex-post intervention.*

1. *If b(T)<0, then at T the intervention is Pareto optimal, that is $ICC(T) > 0$. In this case, the policy keeps the peripheral economy indifferent and makes investors strictly better off.*

2. *The length of intervention is limited. Specifically, there exist $T^E < \bar{T}$, where $\bar{T}$ is the time a steady state would be reached, that is $\dot{b}(\bar{T}) = 0$, such that $ICC(T^E) = 0$ and $ICC(t) < 0$, $\forall t \geq T^E$, so that the incentive compatibility constraint is not satisfied after T^E. Then, at T^E the authority stops the intervention and the peripheral economy defaults.*

See Appendix A.5 for the proof.

Proposition 5 states two important results. The first one is that at the moment of intervention investors are better off than if the government was left to default. Hence, the intervention is Pareto improving.[7] The benefit of the intervention stems from solving the *non-exclusivity externality*. The second one relates to the length of the intervention and answer the question: for how long investors are better off? The length of intervention depends on how the cost for taxpayers grows with respect to the benefit. The statement (2) implies that the cost increases faster so that the intervention is always bounded in time. We can provide an intuition for this result. Recall that the cost of the intervention, $G(\cdot)$, is financed by investors, and it is proportional to the distance between

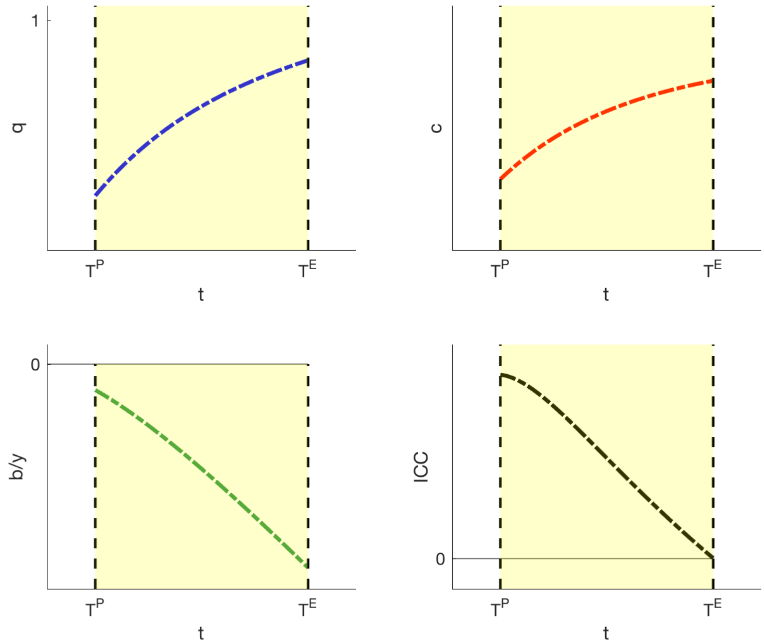
[7] Recall that by construction the policy leaves the peripheral country indifferent. Hence, since investors are better off, then the policy is Pareto improving.

the policy price $q^P(\cdot)$ and the market price $q(\cdot)$, since that distance is also the expected loss on each unit of new bond financing. Now, if $q^P(\cdot)$ is low enough, then the cost of the intervention is relatively small and, therefore, investors' intervention gain that comes from delaying default exceeds its fiscal cost. On the contrary, if the policy price $q^P(\cdot)$ is too high, the fiscal cost of default might exceed the benefit and the authority needs to stop the intervention since it is not anymore Pareto improving. But recall also that by Proposition 4 the policy price is always increasing, which means that to keep the peripheral country as well off, the authority needs to offer continuously better condition. Soon enough the fiscal burden for investors become large enough that the intervention is not anymore beneficial for them, the policymaker stops the policy, and the government defaults.

This mechanism reveals an interesting balance of power. If the peripheral country has large incentives to default, then the authority is forced to offer a high bond price, which is very costly for taxpayer and the intervention will be very short. On the other hand, if the peripheral country has small incentives to default, the intervention is relatively cheap and the investors' are happy to finance the government for a longer time waiting for the good output outcome to realize.

Figure 1.3 plots the dynamics of the bond price $q(t)$, government's assets $\frac{b(t)}{y_L}$ and consumption $c(t)$, and of the function $ICC(t)$ that measures the marginal gain of the policy intervention for investors. When the time of intervention T^P arrives, the policymaker offers an upper sloping price for the government's bond. Better borrowing conditions for the government are welcomed by long-term investors that would have otherwise lost part of their investment due to the upcoming default. At this conditions the government is happy to stay in the market, continuing to borrow and increasing its consumption, while investors are better off since they will continue to receive the interest repayments and default is at least delayed. If the recession is long-lasting, so that the country does not jump to a better state of the economy, the policy continues but become more and more costly for investors. At the time T^E the benefit of the intervention is exactly counterbalanced by that cost, there is no anymore marginal gain for investors to continue to finance the policy, and, therefore, the policymaker stops the intervention, and the country defaults.

FIGURE 1.3: Bond Price and Debt before Default



Note: this graph plots the competitive equilibrium path of the bond price (top-left panel), level of asset as a fraction of output (bottom-left panel), peripheral economy's consumption (top-right panel) and function ICC (bottom-right panel) as a function of time (x-axis).

The setting provided in this section, in which as soon as default time arrives and the market for bond disappears the policymaker intervenes and imposes its condition, is extremely useful to understand how the policy works. However, this setting does not take into account the market reaction to the policy announcement. We tackle this issue in the next section.

Discussion on policy implementation The policy design that we proposed is instrumental to the purpose of highlighting the pecuniary externality at the heart of the problem. Our policy is indeed a transfer from existing creditors to new investors that underwrite bonds on the primary market in the current period. It is this transfer that, by decreasing the interest rates for the borrowing government, gives incentives to the government to borrow further. A subsidy on investment is a policy that is not controversial as it can be easily implemented using market instruments. More controversial is be the ability of the policymaker to tax bondholding, in particular when bondholding are dispersed and held internationally. This casts some doubts on the feasibility of the intervention, in particular because absent the possibility to tax bondholders in a distortionary manner, if anticipated, the policy can be ex-ante detrimental, as we show in the next section.

Once the mechanism is clear, it is then easy to think to alternative arrangements that can lead to the same outcome. Similarly to our policy, we can think that a credit line is provided directly by an international institution that finances the intervention through taxes. Alternatively, we could think to more nuanced contractual arrangements. For example, we can think to a complex seniority structure that, after some debt level, gives progressively increasing seniority to new bondholders, or equivalently that debt is progressively restructured conditional on the country continuing to borrow from the market.

1.6 Ex-Ante Analysis

The previous section was useful to show that in our framework there is a scope for policy intervention and to explain what are the characteristics of the policy when it is in place. The government may default because the bond price offered by the market is too low and existing investors cannot coordinate to provide a better price. We proved that in this case a policymaker that internalizes the interests of existing creditors has always incentive to intervene ex-post and to extend credit to the government, thus eliminating the *non exclusivity externality*. We also proved that, however, the policy is limited in time, since investors' gain from providing additional financing declines to zero. Nevertheless, ex-ante the market reacts to the existence of the policy, since the bond price will incorporate the benefits of future policy intervention. In this section we show that: (i) ex-ante the bond price generally increases ; (ii) as a consequence, the endogenous debt level at which the country would default, is higher than absent policy.

Ex-ante Market Bond Price. First, we investigate how the value of a bond changes when the policy intervention, in the form we have explained in the previous section, is common knowledge. Investors have perfect information about the details of the policy; hence, they are perfectly able to compute the bond value accounting for the intervention. We assume that the policy is financed by a linear combination of a lump-sum tax, which taxes each investor i independently of asset holdings, and a proportional tax per-unit of asset. We can then define the aggregate tax revenue $T(t)$ as

$$T(t) \equiv \int_i a^i(t)\tilde{\tau}(t,\alpha)di + \int_i \tau(t,i,\alpha)di \tag{1.32}$$

where $\tilde{\tau}(t,\alpha)$ is the tax per unit of asset and $\tau(t,\alpha)$ is the lump-sum tax per agent-investor i. The policy rule is indexed by a parameter α, which is the ratio of the tax revenue collected through the proportional tax over the total tax revenue (notice, that we do not impose restriction on α, which can then be greater than one). We restrict attention to symmetric equilibria, hence $a^i(t) = a(t)$ and $\tau(t,i,\alpha) = \tau(t)$. Since the intervention is balanced budget, it must be $G(t) = T(t)$. This implies $\tilde{\tau}(t,\alpha) = -\alpha G(t)/b(t)$ and $\int_i \tau(t,i,\alpha)di = (1-\alpha)G(t)$.

Denote the time of effective intervention with T^P.[8] The value function of an investor who holds $a(t)$ units of bonds and carry them until maturity is equal

[8] As it will become clear in this section the time of intervention when the market reacts to the policy announcement, the authority will intervene at a time T^P is in general different than the default time without intervention, T.

to:

$$V\left(a(t)\right) = \int_t^{T^E} [-a(s)\tilde{\tau}(s,\alpha) - \tau(s,\alpha) + (\rho+\delta+\lambda)a(s)]e^{-(\rho+\delta+\lambda)(s-t)}ds$$
$$+ V(a(T^E)|q(T^E))e^{-(\rho+\delta+\lambda)(T^E-t)}.$$

In equilibrium investors must be indifferent between holding a bond until maturity or selling a bond on the secondary market, then it must be $q(t|\alpha) = V'(a(t))$, where we use the notation $q(t|\alpha)$ to indicate the bond price when an intervention financed with a fiscal policy α is in place. The bond price reads

$$q(t|\alpha) = \int_t^{T^E} [-\tilde{\tau}(s,\alpha) + (\rho+\delta+\lambda)]e^{-(\rho+\delta+\lambda)(s-t)}ds + q(T^E)e^{-(\rho+\delta+\lambda)(T^E-t)}. \tag{1.33}$$

where $q(T^E) = \frac{\phi\lambda}{r+\lambda}$. Three remarks are worth making. First, the bond price depends upon the time T^P in which the intervention starts. This is because investors know that the government will finance its policy levying taxes starting from T^P. However, T^P is an equilibrium object which is a function of α and the initial level of debt $b(0)$. Second, investors internalize the intervention and they understand that the interest repayment will continue until the time in which the intervention stops, T^E. Therefore, ex-ante the bond value appreciates, because the policy will delay default. Third, notice that the value of a bond depends on the extent to which taxation is affected by individual portfolio decisions. The term $\int_t^{T^E} \tilde{\tau}(s,\alpha)e^{-(\rho+\delta+\lambda)(T^E-t)}ds$ captures this effect. Specifically, taxation affects the value of bond only on the way the individual tax varies with individual bond holdings, $a(t)$. The dependence of the tax on t captures the fact that the total amount of tax revenue required to finance the intervention varies with time, since the total revenue needs to equate the planner loss, as described in equation (1.23).

Ex-Ante Symmetric Rational Expectation Equilibrium. We are now ready to define the ex-ante symmetric equilibrium of our economy when the intervention is anticipated and prior to the jump to the high income state.

Definition 4. An Ex-Ante Symmetric Rational Expectation Equilibrium is: a time of policy intervention T^P, a time of end of policy T^E, a market bond price $\{q\,(t|\,\alpha)\}_{t=0}^{T^E}$, a policy bond price $\{q^P(t)\}_{t=T^P}^{T^E}$, a tax rule α, a gross subsidy $\{G(t)\}_{t=T^P}^{T^E}$ a saving policy $\{b(t)\}_{t=0}^{T^E}$, such that

(i) Given α, and associated gross subsidy $\{G(t)\}_{t=T^P}^{T^E}$ the bond market price satisfies (1.33) from $t \in [0, T^E]$.

(i) the government solves problem (1.14) taking as given the market bond price $\{q\,(t|\,\alpha)\}_{t=0}^{T^E}$ for $t < t \leq T^P$, and the policy bond price $\{q^P(t)\}_{T^P}^{T^E}$ for $T^P \leq t < T^E$.

(i) the monetary/fiscal authority:

- intervenes at T^P if $T^P < T^j$ and solves its problem in (1.26)-(1.28);
- stops the intervention at T^E if $T^E < T^j$;
- follows the policy rule in (1.32) and balance its budget as in (1.23)

(i) bond markets clear.

It is important to understand that with the policy in place, the government does not choose anymore the optimal time of default T, but it chooses the optimal time of intervention T^P. However, since the intervention makes the government indifferent between default and repayment, it does not affect the scrap value in problem 1.14, and the government problem remains identical as before, with the difference that the bond market price at intervention $q(T^P|\alpha)$ is a function of the policy rule α and it will be endogenous to the gain in time from the intervention. The following result characterizes the optimal intervention and allow us to pin down he equilibrium.

Proposition 6. *Characterization of Ex-Ante Equilibrium. In the ex-ante symmetric rational expectation equilibrium described above, at the time of intervention T^P, it must be :*

$$q\left(T^P|\alpha\right) = q^P(T^P).$$

See Appendix A.6 for the proof.

1.7 Welfare

As the equation (1.33) displays, the ex-ante effects on the equilibrium bond price depend on how the individual taxation is linked to the amount of individual asset holdings. Hence, the ex-ante welfare implications of the policy are tightly related to the fiscal rule, α, implemented to finance the intervention. In this section, first we illustrate the ex-ante equilibrium in the case the policy is fully financed with a non-distortionary lump-sum tax ($\alpha = 0$). Then we generalize our findings for a generic α. .

1.7.1 Non-distortionary Taxation

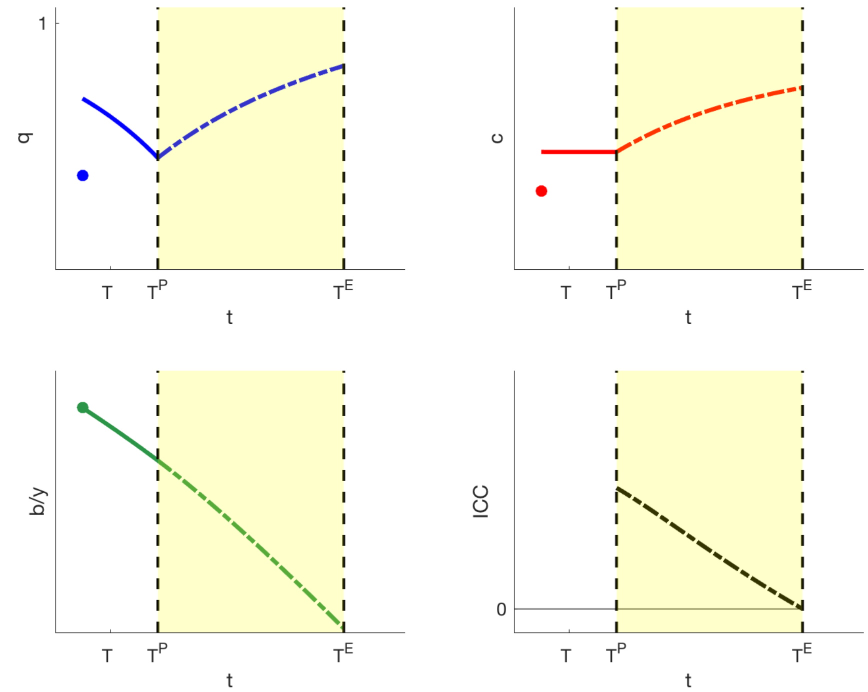
Let first assume that the monetary/fiscal authority finances the policy intervention entirely with a lump-sum tax. This is equivalent to set $\alpha = 0$. This tax is not distortionary. In fact, as equation (1.33) shows, the lump-sum tax does not affect an investor's bond evaluation when the policy is announced. However, the fact that the cost of the policy is not internalized by the bond price generates an over-valuation of the bond which results in an ex-ante loss for investors. When the policy is accounted for, the investor prices the bond considering that the policy generates a delay of default. The bond then becomes more appealing and its value appreciates. Nevertheless, investors will have to bear the entire cost of the policy. Figure 1.4 displays the dynamics of the main economic variables. The top-left panel represents the bond price. The dot represents the value of the bond in a model without intervention. The solid line represents the ex-ante bond prices that account for the existence of the policy. The policy, then, improves the credit conditions faced by the peripheral country, which can now borrow at a lower interest rate. As a consequence, ex-ante the government can increase its consumption level by borrowing at a higher pace, as displayed in the top-right plot. If the recession is long lasting, eventually the bond price devaluates until, at time T^P, the policy will be implemented. Notice that the policy announcement per-se delays the policy implementation of a quite large time since, absent market reaction as displayed in Figure 1.3, the authority would have intervened at time T. Hence, the intervention following the announcement occurs at a quite large debt-to-income level, and, by Proposition 5 the ex-post net gain from the intervention is lower, as measured by the low value of the ICC at the moment of intervention. Also, from the bottom-left figures it is clear that the existence of the policy makes a higher level of debt more sustainable, since the endogenous debt limit is $b(T^P) < b(T)$.

1.7.2 Distortionary taxation and Pareto Improving Policies

The example with the policy financed with a lump-sum tax helps to understand an important feature of the ex-ante welfare effects of the policy: the bond price appreciation that stems from the fact that taxes are not internalized in the bond price, acts as an implicit fiscal transfer from investors to the peripheral country; the higher the bond price appreciation, the higher is the magnitude of that transfer. In this subsection we formalize this concept, allowing for the possibility that taxation is distortionary ($\alpha > 0$).

In order to properly address the welfare effect of the policy, suppose that the policy is announced at time 0, and that the government has a corresponding outstanding initial level of debt $b(0) \leq 0$. We define the ex-ante welfare gain of a representative investor that holds all the initial stock of debt $b(0)$ and underwrites every new bond issuance as $\Delta V(\alpha, b(0))$. Similarly, we define with $\Delta W(\alpha, b(0))$ the ex-ante welfare gain of the borrowing government.

FIGURE 1.4: Bond Price and Debt before Default



Note: this graph plots the ex-ante competitive equilibrium path of the bond price (top-left panel), level of asset as a fraction of output (bottom-left panel), peripheral economy's consumption (top-right panel) and ICC function (bottom-right panel) as a function of time (x-axis). The dots represent the value of the variables absent policy.

The notation makes explicit that the two measures of welfare are a function of the relevant fiscal rule α and of the initial stock of debt at announcement $b(0)$. The first step of our analysis is to characterize the link between the fiscal rule α and these welfare measures. The gain from the policy for the representative investor is the sum of two components:

$$\Delta V(\alpha, b(0)) = -b(0) \int_{T}^{T^E} (r + \lambda(1 - \phi))e^{-\delta T} e^{-(r+\lambda)(s-t)} ds + \\ - (1 - \alpha) \int_{T^P}^{T^E} G(s) e^{-(r+\lambda)(T^E - t)} ds. \tag{1.34}$$

The first term is the gain from the delay in default brought about by the policy but not internalized before the announcement. To the extent that $T^E > T$, this term is positive. The second term is the cost brought about by financing part of the intervention with lump-sum tax. This term is negative as long as $\alpha < 1$. If we restrict attention to $b(0) = 0$, which correspond to the case where investors knew about the policy since the onset and $\alpha \geq 1$, we are able to provide the following characterization:

Proposition 7. *Let $q(T^P)$ be the bond price at intervention, $b(T^P)$ be debt at intervention, $\Delta W(\alpha, 0)$ the welfare gain of the borrowing country and $\Delta V(\alpha, 0)$ the welfare gain of the representative investor, then for $\alpha \geq 1$*

- *$q(T^P)$ and $W(\alpha, 0)$ are monotonically decreasing in α,*
- *$b(T^P)$ and $V(\alpha, 0)$ are monotonically increasing in α.*

See Appendix A.7 for the proof.

Proposition 7 states a very important result. The monetary/fiscal authority can use the fiscal policy α as redistribution instrument between investors' welfare and the peripheral country's welfare. Financing the intervention by imposing heavy taxes to bond holding depresses bond prices and the country's welfare in favour of investors' welfare. On the contrary, by attenuating the dependence of the tax upon bond holding, the policymaker creates an over valuation of the bond price which diminishes investors' welfare in favour of the country's welfare by creating an implicit fiscal transfer to the latter. This proposition indeed shows that α is actually a measure of the implicit transfer from investors to the country.

The second step is to characterize the set of Pareto improving interventions.

At the limiting case in which the fiscal rule aggressively taxes bond holding ($\alpha = \bar{\alpha} > 1$) so that $q(T^P) = q(T)$, the welfare gain of the peripheral country must be equal to zero. In this case the existence of the policy does not affect the market price before intervention and therefore, its effects are equivalent to the effects of an ex-post policy that takes place at T. Investors will get the entire benefit from the intervention as the amount of the tax revenue collected through the proportional tax in excess of the financing cost, that is $(\bar{\alpha} - 1)G(t)$, is redistributed lump-sum to the investors. On the opposite extreme of the

Pareto set, in which $\alpha = 1$, by (1.34), it must be $\Delta(\alpha, 0) = 0$, and all the benefit from the intervention will go to the peripheral country.

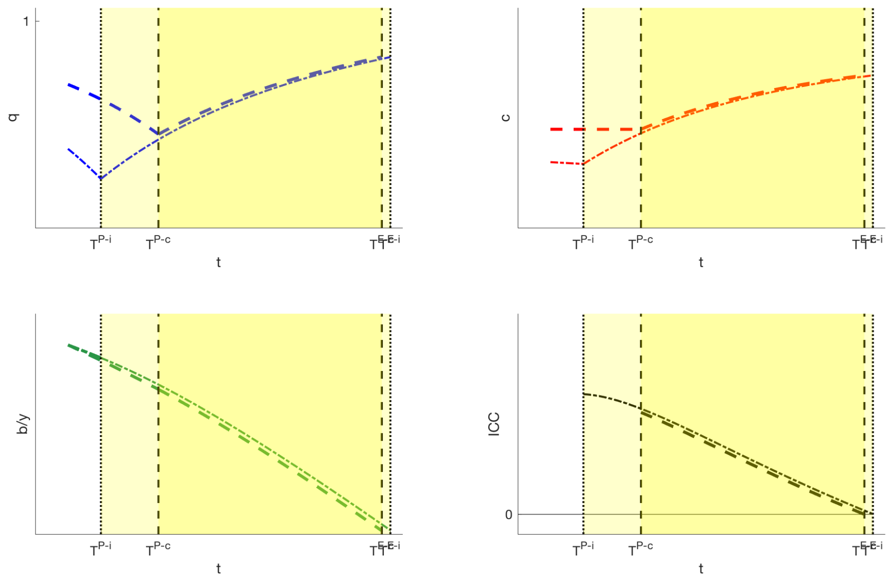
In addition, when α decreases, then, by Proposition 7 investors' gain from intervention decreases and the country's gain increases. That means that all the policy characterized by α such that the investor's gain is still positive are Pareto improving. The following Proposition formalizes this result.

Proposition 8. ***Pareto set.*** *For $b(0) = 0$, there exists a non empty set $\alpha \in [1, \bar{\alpha}]$ of Pareto Improving policies. In particular, $\Delta W(1, 0) > 0$ and $\Delta V(1, 0) = 0$, while $\Delta W(\bar{\alpha}, 0) = 0$ and $\Delta V(\bar{\alpha}, 0) > 0$.*

See Appendix 8 for the proof.

In Figure 1.5 we display the equilibrium paths of the policy that are at the boundary of the Pareto improving set. We show two cases. The first case is characterized by a fiscal policy that taxes aggressively bond holding. In this scenario $\alpha > 1$ and $\Delta W(\alpha)$ is equal to zero, as displayed by the dotted line. As stated in Proposition 8, this scenario is characterized a positive gain for investors and by the smallest, and equal to zero, gain for the peripheral country. The time of intervention in this case is indicated with T^{P-i} and the intervention region is represented with a light shaded area. The second case is characterized by a fiscal policy that taxes less aggressively bond holdings. In this scenario $\alpha = 1$ and the bond market price is higher than the one absent policy, i.e. $\Delta q(b)$ is positive, and equal to $\bar{\Delta} q(b)$, as displayed by the dashed line. This scenario is characterized by the largest gain from the policy for the peripheral country and by a gain for investors equal to zero. The time of intervention in this case is indicated with T^{P-c} and the intervention region is represented with a dark shaded area. The jump in the bond price delays the time of intervention, to T^{P-c} and, increases the country's consumption, and diminishes the ex-post incentive to intervene for investors, as indicated by the ICC panel. Importantly, any fiscal policy that implies a size of the jump between the two cases displayed in the figure represent a Pareto improving policy. Also, the largest is the size of that jump the largest is the redistribution of welfare from investors to the peripheral country. Indeed, the jumps map one-to-one movements in α.

FIGURE 1.5: distortionary taxation and Pareto Improving Policies



Note: this graph plots the competitive equilibrium path of the bond price (top-left panel), level of asset as a fraction of output (bottom-left panel), peripheral economy's consumption (top-right panel), and ICC function (bottom-right panel) as a function of time (x-axis).

Finally, in Table 1.1 we present some summary statistics about the equilibrium dynamics of our model under three different fiscal policies. In the first column we study an economy such that that the implied price appreciation due to accounting for the existence of the policy is zero, so that that fiscal rule implies a Pareto improving policy that is optimal for investors. This case should be interpreted as a policy that aggressively taxes bond holdings. In the second column we study an economy such that that the ex-ante implied price appreciation is $\bar{\Delta}q(b(0))$, so that that fiscal rule implies a Pareto improving policy that is optimal for the peripheral country. This case should be interpreted as a policy that mildly taxes bond holdings. In the third column we study an economy in which the authority imposes a lump-sum tax to finance the policy, and, therefore, the policy does not tax at all bond holdings. The calibration of the parameters of the model is as follow. The recession implies at income y_L

that is 85% lower than in the good state, y_H. The maturity of the bond is δ=0.5. The debt repayment after default, ϕ is set to be equal to 0.95. The interest rate r and the discount rate ρ are equal to 0.02. Finally, the economy starts at period t_0 with zero debt.

TABLE 1.1: Statistics of three different fiscal policies

	Pareto Improving		Non Pareto Improving
	Optimal for Investors	Optimal for Country	Lump-Sum Tax
		Asset Price and Debt	
Spread without policy at $b(0)$	19.29	19.29	19.29
Spread with policy at $b(0)$	19.29	6.96	2.30
Δ Spread	0	-12.33	-16.99
Debt/GDP at Intervention	13.21	29.75	60.12
		Probabilities at Announcement	
Prob. of Intervention	95	87	77
Prob. of Default	69	63	64
		Welfare Gain/Loss from Policy	
Welfare Investors (Cons. Equiv)	0.64	0	-1.40
Welfare Country (Cons. Equiv)	0	0.66	0.70

In our economy at the initial time the spread, measured as the annualized differerence between the return on the sovereign bond and the risk free rate, is about 19 percent. The second and third row displays the effect of the policy on the sovereign bond interest rate at that debt level. By construction, the case in which the policy is optimal for investor is designed such that there is no appreciation on the value of the bond. Hence, the annualized spread in the ex-ante equilibrium remains 19 percent. In the second scenario, the authority taxes lightly bond holding and the spread becomes 7 percent. This change reflects a bond price jump equal to $\bar{\Delta}q(b(0))$. Finally, if the authority finances the policy via lump-sum taxes, the spread decline drastically to 2.3 percent. Hence, the existence of the policy creates a large bond evaluation just because investors have incentive to demand sovereign bonds. The different fiscal policy and the associated changes in the bond prices imply different moments in which the intervention will take place: the lower the bond price jump, the earlier the authority will intervene. Since, if the recession is long lasting, the peripheral county's debt increases with time, it follows that different fiscal policies also imply different degree of debt sustainability before the intervention. As the forth row shows, if the policy does not affect bond prices, the authority will intervene early, that is at low level of debt-to-GDP ratios (13 percent). If the fiscal policy is financed with lump-sum taxes, the large increase in market valuation of the bond implies that the market will be able to support much large sovereign debt, up to 60 percent in terms of GDP. As the second panel shows, we can compute the probability of intervention, and of default, in each scenario. Recall, that if the exogenous process for income jumps to the higher state, default will never occur and therefore there is no need of intervention. We can then compute how different policies affects the probability of intervention and default. We can notice that if the announcement does not alter the bond price intervention is very likely (95 percent). When the fiscal policy induces bond appreciation, the market, as already explained, can sustain the sovereign borrowing for longer. The probability for intervention falls to 87 and 77 percent for the other two scenarios. Intuitively, by taxing bond holding non aggressively, the authority can induces bond appreciation and can buy time, hoping that a good realization of income realizes before an intervention is needed. Nevertheless, notice that the default probability do not change drastically across the three scenarios. This is because an intervention that starts at larger level of debt is shorter, since investors' gain are reduced. The third panel shows the ex-ante welfare gain/loss from the intervention, in terms of consumption equivalent. In the first scenario, the country does not benefit from the policy. This is intuitive because the policy is designed to make the peripheral country as well off as its default value. Since in the first scenario the existence of the policy does not alter the market outcome, then the peripheral must not benefit from the policy. On the contrary, investors are better off, as much as 0.64 percent in consumption equivalent, because the intervention is able to address the *non-exclusivity externality*. It can be inferred that for our calibration this externality is quite costly in terms of welfare. This scenario is at one extreme of the spectrum of Pareto improving policies, since the peripheral country is unaffected by the policy intervention. The second scenario also delivers not surprisingly results. In fact, in that case the fiscal policy is designed to create bond price jump equal to $\bar{\Delta}q(b)$, which, by Proposition 8, is defined to generate investors' gain equal to zero. In this case, the authority mildly taxes bond holding, the bond price at announcement slightly jumps, and therefore the market outcome is affected. The bond appreciation allows the peripheral country to borrow at better condition even before the intervention. Hence, it can enjoy higher consumption and its welfare increases. This is the other extreme of the spectrum of Pareto improving policies, in which now investors are unaffected by the policy: the gain obtained by eliminating the *non-exclusivity externality* are completely transferred to the peripheral country. Finally, the last column shows that a lump-sum taxation induces a so large bond appreciation that investors are worse off from the intervention. This is because the policy creates ex-ante incentives to demand bonds that, in turns, make the policy non beneficial from the investors' point of view.

1.8 Conclusions

This paper highlights the presence of a novel externality in sovereign bond markets, that we label as *non-exclusivity externality*. This externality arises because in a competitive market, in which the ownership of debt is anonymous and dispersed, the equilibrium price of new debt on the primary market might be too low to avoid default, even though preventing default would be in the interest of existing creditors. We then show that a policy that subsidizes the underwriting of new bond issuance by taxing existing bondholders, is ex-post Pareto improving. This is true because, upon default, the benefit from delaying default is always greater than the cost, even though the duration of the policy is limited in time. However, the benefit ex-ante depends crucially on the ability of the policy-maker to tax bond-holdings in such a way that the price ex-ante will reflect the cost of the policy. Absent this possibility, the policy is ex-ante detrimental.

Chapter 2

Credit Failures and Interventions

When asymmetric information in credit markets constrains investment, the quality of public information about firms is vital to sustain investment and economic growth. The aftermath of the 2008 crisis reminded it to us, as rating agencies downgraded around 90% of the structured products they had classified as investment grade prior to Lehman's bankruptcy, casting doubts about the usefulness of ratings to predict creditworthiness. In this paper, we show that when adverse selection is severe: (i) public information affects the investment rate of the economy; (ii) competitive equilibria are constrained inefficient, so regulator interventions can improve on market outcomes, even though they face the same set of constraints; (iii) purely *informational shocks* – that is, shocks to the degree of public information that do not affect other fundamentals of the economy – can generate sharp and persistent contractions in investment and production, that can help explain the prolonged capital and labor unemployment after a crisis.

To make these arguments, we consider an infinite horizon, two-goods version of Philippon and Skreta (2012), with a long-lived household and overlapping generations of firms. At each date, credit markets are segmented: a set of heterogeneous firms is able to invest under full information (they are *transparent*). The others are subject to a lemon problem à la Akerlof (1970), and so are *opaque*. To clarify the effect of purely informational shocks, we let the proportion of each type of firm in the two subsets be the same, so that shifting mass from one subset to the other does not have any impact on fundamentals other than information. Credit is offered by the representative household, via an unmodeled competitive banking sector, and it affects the chances that the firm will be productive in the subsequent period. Conditional on being productive, all firms are identical: they employ capital to produce the only other good in the economy (consumption) under decreasing returns to scale.[1]

We first characterize the steady states of a non-stochastic version of this economy, where firms profits flow back to the household sector. When adverse selection is not severe, competitive allocations are efficient and the economy is at full investment. Public information has no effect on growth rates. An increase in the degree of informational asymmetries brings about equilibria with less than full investment, and there exists a threshold in the severity of the adverse selection problem below which *all* competitive equilibria are constrained inefficient. We prove this constructively, showing that a balanced-budget policy that subsidizes investment and taxes consumption in steady state implements a competitive equilibrium with full investment. In this case, the quality of public information affects the pool of investing firms. In particular: (i) it is positively correlated with aggregate investment and growth; (ii) it does not affect credit spreads because the pool of firms subject to adverse selection is unaffected. The latter is a feature of steady states which will disappear once we consider informational shocks.

Then, we extend the analysis to the case where firms consume their own profits. We derive a necessary and sufficient condition for *local* constrained suboptimality, which roughly states that there exists a local Pareto improving intervention if and only if the percentage increase in the mispricing of the securities issued by the marginal type between the laissez faire equilibrium and the aftermath of the intervention is weakly lower than the household's gain from a marginally higher investment rate.

Finally, we consider the effects on the economy of a one-time unanticipated

[1] Decreasing returns to scale guarantees that productive firms make strictly positive profits, which can cover the sunk investment cost eventually incurred.

informational shock to the quality of public information. The exercise is motivated by the rating crisis of late 2008, but we believe that more general lessons can be learned by it. A sudden drop in the quality of public information generates several effects that are consistent with the data. First, after a short-lived increase, the capital stock plummets and so does the aggregate volume of credit to the economy. Both drops are quite persistent, for instance a 20% drop in the quality of public information may lead to a capital stock that is 15% lower than the steady state level after 60 quarters.

Credit market dynamics are of particular interest. They are characterized by two phases. In the first phase, shortly after the shock, credit to opaque firms expands, and interest rates fall. This is due to the lower overall credit granted (driven by the drop in demand by high quality firms, a larger proportion of which is now opaque), which increase the expected value of investment by opaque firms and so increases the threshold for investing firms within this pool. In the second phase, as the public information channels are gradually restored, credit to opaque firms fall and interest rates recover.

Overall, we believe that studying the role of public information in economies where credit is subject to asymmetric information is not only realistic, but also important to generate predictions consistent with the facts and to justify regulatory interventions. We provide necessary and sufficient conditions for interventions to be beneficial, and we implement efficient allocations by means of investment subsidies, consumption taxes and decentralized markets. We show that purely informational shocks do justify interventions, and generate prolonged periods of low investment and low growth, despite the absence of any productivity shock.

The paper has the following structure: Section 2.1 presents the model economy and its properties; Section 2.2 characterizes the set of competitive equilibria and their efficiency properties under full information first and then under asymmetric information; Section 2.3 studies the constrained efficiency properties of the equilibria, 2.4 studies the effect of a pure informational crisis; Section

2.5 concludes.

2.1 The Model Economy

In this section we discuss our benchmark model economy. Time is discrete and lasts forever, $t = 0, 1, 2, ..., \infty$. The economy is populated by a unit measure of identical households and overlapping generations of entrepreneurs. Households are infinitely lived, while entrepreneurs live for two periods. Each period a new generation of entrepreneurs of measure one is born, so that two generations of entrepreneurs are alive at each point in time. There is a single final good that can be either consumed or transformed in capital using a one-to-one technology.

2.1.1 Households

The representative household has lifetime utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t u(C_t) \tag{2.1}$$

where $u(\circ)$ is an increasing and strictly concave utility function that satisfies the Inada conditions and $\beta \in (0,1)$ is the rate of time preference. The household faces the following budget constraint:

$$C_t + B_t + I_t \leq R_{t-1}B_{t-1} + P_t K_{t-1} + \chi \Pi_t. \tag{2.2}$$

Each period, the household rents his initial capital holdings K_{t-1} at a unit price P_t, receives gross interests $R_{t-1}B_{t-1}$ from bond holdings, and profits $\chi\Pi_t$, from holding a fraction $\chi \in [0,1]$ of shares in the productive sector. The household uses the proceedings to consume C_t and invest either in capital production I_t or in a fully diversified portfolio of loans to the entrepreneurs B_t. Capital depreciates at a rate δ, and the law of motion for capital yields

$$I_t = K_t - (1-\delta)K_{t-1}. \tag{2.3}$$

Every period households, taking prices R_t, P_t as given, choose C_t, K_t, B_t to maximize (1) subject to (2), (3) and the transversality conditions:

$$\lim_{T\to\infty} \beta^T u'(c_T)B_T = 0,$$
$$\lim_{T\to\infty} \beta^T u'(c_T)K_T = 0.$$

Optimization yields the Euler equation:

$$E_t\left[\frac{\beta u'(C_{t+1})}{u'(C_t)}\right] = \frac{1}{R_t}. \tag{2.4}$$

together with the no-arbitrage condition

$$E_t\left[\left(\frac{\beta u'(C_{t+1})}{u'(C_t)}\right)(1+P_{t+1}-\delta)\right] = 1. \tag{2.5}$$

2.1.2 Entrepreneurs

The technology to produce the final good is run by overlapping generation of risk neutral entrepreneurs. Entrepreneurs live for two periods, but they can be productive only in the second period of their lives. Each generation consists of a continuum of agents with aggregate mass equal to one, who are born with heterogeneous types $\theta \in [0,1]$, drawn according to the distribution function $F(\theta)$. A type-θ entrepreneur who is born at date t will be productive at $t+1$ with probability $\underline{p}(\theta) \in [0,1]$. If productive, she runs a type-independent decreasing return to scale technology that uses capital as input:

$$y_t = k^{\alpha}.$$

We order the type space so that $\underline{p}(\theta) > \underline{p}(\theta') \iff \theta > \theta'$. Crucially, young entrepreneurs at date t can invest in order to increase the probability of being productive at $t+1$ up to $\bar{p}(\theta) > \underline{p}(\theta)$. We follow Philippon and Skreta (2012) and make the following assumption

Assumption 1. *Gains from investment are type-independent:* $\bar{p}(\theta) - \underline{p}(\theta) = \zeta$.

Investment requires a fixed input of x units of date t consumption good. Entrepreneurs are born with no endowment and need to finance investment by borrowing from the household. In the second period of their lives they are matched with a random household to whom they give a fraction $\chi \in [0,1]$ of the profits and consume the rest. They get utility only from consuming in the second period of their lives and from the utility of the households they are matched with. The problem of a type-θ entrepreneur conditional on the information set $\Im$ can be cast as

$$\Pi_t(\theta|\Im) = \max_{k_t, \mathbb{1}_t} \left(\underline{p}(\theta) + \mathbb{1}_t \zeta\right) \left[k_t^\alpha - E_t[P_{t+1}]k_t - \mathbb{1}_t R_t^L(\theta|\Im)x\right] \tag{2.6}$$

where $\mathbb{1}_t$ is an indicator variable that takes value 1 if investment is undertaken and 0 otherwise, while $R_t^L(\theta|\Im)$ is the lending rate on the credit market conditional on the information available. Entrepreneurs maximize (2.6) taking prices $E_t[P_{t+1}]$ and $R_t^L(\theta|\Im)$ as given. Optimality requires

$$E_t[P_{t+1}] = \alpha k_t^{\alpha-1}, \tag{2.7}$$

$$\mathbb{1}_t(\theta|\Im) = \begin{cases} 1 & \text{if} \quad \zeta\left[(1-\alpha)k_t^\alpha\right] > R_t^L(\theta|\Im)\bar{p}(\theta)x \\ \{0,1\} & \text{if} \quad \zeta\left[(1-\alpha)k_t^\alpha\right] = R_t^L(\theta|\Im)\bar{p}(\theta)x \\ 0 & \text{if} \quad \zeta\left[(1-\alpha)k_t^\alpha\right] < R_t^L(\theta|\Im)\bar{p}(\theta)x \end{cases} \tag{2.8}$$

Notice that, despite each household faces idiosyncratic uncertainty since it might be matched with an unproductive entrepreneur, there is no uncertainty about aggregate productivity in the economy. This implies that in the formulation of the profit function (2.6), entrepreneurs does not need to consider the household discount factor. Indeed, since markets are complete, each household is able to fully diversify its idiosyncratic risk. It follows that each household will get an equal share of aggregate profits.

2.1.3 Information structure

Entrepreneurs know their types; however, their types are private information. They are publicly observable only with probability λ. Define the information set $\Im$ as

$$\Im \equiv \begin{cases} 1 & \text{if type is publicly observable} \\ 0 & \text{otherwise,} \end{cases}$$

then λ is the probability that $\Im = 1$. It follows that by the law of large numbers, a fraction λ of each θ-type entrepreneur is 'transparent' and a fraction $1 - \lambda$ is 'opaque'. Credit markets are segmented: absent costless and credible signaling opportunities, the investment decision of 'opaque' entrepreneurs will be affected by asymmetries of information, while the 'transparent' entrepreneurs will invest under full information.

In the first part of the paper we characterize the steady state of the economy and perform comparative statics under the assumption that λ is constant and time invariant. In the second part, we use the variable λ to introduce *informational shock*, that is shocks which alter the degree of information but not the fundamentals of the economy. We assume that λ follows an $AR(1)$ process of the form

$$\lambda_t = \frac{e^{\xi_t}}{1 + e^{\xi_t}} \quad \xi_t = (1 - \rho)\mu + \rho\xi_{t-1} + \epsilon_t \tag{2.9}$$

$$\text{for } \rho \in [0, 1),\ \mu \geq 0 \text{ and } \epsilon_t \sim N(0, \sigma_\epsilon^2).$$

Shocks to λ are the sole driver of fluctuations in our model. Summing up, our economy has five endogenous quantities at each date t: household's consumption C_t, household's investment in capital I_t; household's deposits B_t, firm's investment policy $\mathbb{1}_t(\theta|\Im)$. The endogenous prices at date t are: the price of capital P_t, the lending rate $R_t^L(\theta|\Im)$ and the deposit rate R_t. Notice that the lending rate can vary depending on entrepreneurial types $\theta \in \Theta$ in equilibrium only if either (i) types are publicly observable (i.e. $\Im = 1$), or (ii) they can be credibly signalled. After defining the notion of competitive equilibrium, we consider the full information case first, to provide a benchmark, and then we will introduce adverse selection.

2.2 Competitive equilibrium

The competitive equilibrium of the model economy is described as follows.

Definition 5. *A competitive equilibrium consists in prices* $\{R_t, R_t^L(\theta|\Im), P_t\}_{t=0}^{\infty}$ *and allocations* $\{C_t, K_{t-1}, k_{t-1}, B_t\}_{t=0}^{\infty}$ *such that:*

- *Households maximize their utility given prices* $\{R_t, P_t\}_{t=0}^{\infty}$
- *Entrepreneurs maximize profits given prices* $\{R_t, P_t, R_t^L(\theta|\Im)\}_{t=0}^{\infty}$
- *Prices clear the markets:*

$$
\begin{aligned}
K_t &= \left(\int_0^1 (\underline{p}(\theta) + \mathbb{1}_t(\theta)\zeta) f(\theta) d\theta \right) k_t \\
C_t + I_t + B_t &= \left(\int_0^1 (\underline{p}(\theta) + \mathbb{1}_{t-1}(\theta)\zeta) f(\theta) d\theta \right)^{1-\alpha} K_{t-1}^{\alpha} \\
B_t &= x \left[\lambda \int_0^1 \mathbb{1}_t(\theta|1) f(\theta) d\theta + (1-\lambda) \int_0^1 \mathbb{1}_t(\theta|0) f(\theta) d\theta \right]
\end{aligned}
$$

2.2.1 Full information

We first restrict attention to the competitive equilibrium of the economy under full information. This corresponds to the case where the types of every entrepreneur are public observable with probability $\lambda = 1$. Under full information, absence of arbitrage opportunities on the credit market requires that every entrepreneur of type θ is charged the lending rate

$$R^L_{t-1}(\theta|1) = \frac{R_{t-1}}{\bar{p}(\theta_t)}, \tag{2.10}$$

which takes into consideration that, by the law of large numbers, idiosyncratic risk can be fully diversified and only a fraction $\bar{p}(\theta)$ of the loans will be paid back. Substituting (2.10) in (2.8), it follows that the decision to invest under full information is type-independent and given by

$$\mathbb{1}_t = \begin{cases} 1 & \text{if} \quad R_t x < \zeta(1-\alpha)k_t^\alpha \\ \{0,1\} & \text{if} \quad R_t x = \zeta(1-\alpha)k_t^\alpha \\ 0 & \text{if} \quad R_t x > \zeta(1-\alpha)k_t^\alpha, \end{cases} \tag{2.11}$$

The condition above tells us that investment is undertaken if it features net positive present value, that is, if the discounted additional revenue from investment $\zeta(1-\alpha)k_t^\alpha / R_t$ is greater than the cost x. However, notice that for a given level of aggregate capital K_t, k_t is endogenous to the investment decision $\mathbb{1}_t$. Therefore, we cannot rule out the possibility of multiple equilibria.

Proposition 9. *If $\lambda = 1$, the competitive equilibrium is characterized by equations (2.2),(2.3),(2.4), (2.5),(2.6),(2.7),(2.8),(2.11). These 8 equations, fully determine the dynamics of prices R_t, $R^L_t(\theta)$, P_t and quantities $C_t, I_t, K_t, B_t, \mathbb{1}_t$*

Steady state characterization

Equations (2.4),(2.5),(2.7) together imply that at the steady state $R = 1/\beta$, and $k = \left[\frac{\alpha\beta}{1-\beta(1-\delta)}\right]^{\frac{1}{1-\alpha}}$. Since we are interested in deviation from full investment, and we want to restrict attention to pure strategies, we make the following assumption that we maintain for the rest of the paper:

Assumption 2. Investment has net positive present value at the steady state:

$$x < \beta\zeta(1-\alpha)\left[\frac{\alpha\beta}{1-\beta(1-\delta)}\right]^{\frac{\alpha}{1-\alpha}}$$

Proposition 10. *Under assumption* (2) *the steady state is unique and features full investment. At the steady state the economy achieves the first best.*

Proof. The first statement follows immediately from equation (2.11). Pareto optimality follows from realizing that the investment condition (2.11), is the same of an unconstrained social planner that maximizes household utility subject to the aggregate resource constraint. □

2.2.2 Adverse selection

We now extend our benchmark economy to consider adverse selection. To this end, we assume $\lambda < 1$, and we restrict attention to the case where a positive fraction of entrepreneurs are affected by asymmetries of information in the credit market. Let focus on the 'opaque' entrepreneurs, since we dealt with credit markets under full information in the previous section.

Lemma 4. *When entrepreneurial types are private information, two properties of interest rates must hold:* i) *there can only be a pooling rate of interest.* $R_t^L(\theta|0) = R_t^L$, $\forall\theta \in [0,1]$. ii) *Whenever type* θ' *invests, so do all* $\theta \in [0, \theta')$.

Proof.

1. First, there can only be a pooling rate of interest. To see why, consider the profit function (2.6) conditional on investment - i.e., with $\mathbb{1} = 1$. Revenues are equal to $\zeta(1-\alpha)k^\alpha$, independently from the entrepreneur's type θ. Costs are $\bar{p}(\theta)(1 + R_{t-1}^L)x$, and they increase with R_{t-1}^L. So, if two or more rates are quoted, everyone would choose the lower rate: screening of types is impossible.[2]

[2] Technically, the loan rate r might still depend on θ if entrepreneurs resort to *random contracts*, i.e. contracts that consist in both a loan rate, and a probability with which the loan is granted strictly between zero and one for all $\theta \in \Theta \setminus \{\underline{\theta}\}$. We do not consider such contracts

2. Second, since (i) R_t^L is independent from θ, and (ii) costs increase in $\bar{p}(\theta)$, it follows that whenever type θ' invests, so do all $\theta \in [0, \theta')$.

□

As a result, the set of investing types is fully characterized by a threshold $\hat{\theta}$, where

$$\hat{\theta}_t \equiv \sup\{\theta \in [0,1] \mid \Pi_t(\theta|\Im = 0, \mathbb{1}_t = 1) \geq \Pi_t(\theta|\Im = 0, \mathbb{1}_t = 0)\}. \tag{2.12}$$

By the law of large numbers, the fraction of productive firms who invest subject to informational asymmetries is exactly equal to $F(\hat{\theta})$. The pooling lending rate solves:

$$R_t^L = \frac{R_t}{\mathbb{E}[\bar{p}(\theta)|\theta \leq \hat{\theta}_t]} \tag{2.13}$$

where $\mathbb{E}[\bar{p}(\theta)|\theta \leq \hat{\theta}_t] \equiv \int_0^{\hat{\theta}} \bar{p}(s)\frac{f(s)}{F(\hat{\theta})}ds$. By the definition of $\hat{\theta}$ in (2.12), the threshold $\hat{\theta}_t$ can be derived from the indifference condition of the entrepreneurs as

$$\underbrace{\left[\frac{\bar{p}(\hat{\theta}_t)}{\mathbb{E}[\bar{p}(\theta)|\theta \leq \hat{\theta}_t]}\right]}_{\text{Information premium}} R_{t-1}x = \zeta(1-\alpha)k_t^{\alpha} \tag{2.14}$$

Proposition 11. *The competitive equilibrium is characterized by equations* (2.2),(2.3),(2.4), (2.5),(2.6),(2.7),(2.8),(2.11), (2.14). *These 9 equations, fully determine the dynamics of prices R_t, $R_t^L(\theta)$, P_t, the pooling rate R_t^L, quantities $C_t, I_t, K_t, B_t, \mathbb{1}_t$ and threshold $\hat{\theta}_t$*

because: (i) our qualitative results do not change if we allow for such contracts. In particular, random contracts never implement full investment because all types higher than $\underline{\theta}$ have to choose a probability of investment strictly less than one; (ii) we would need to introduce incentive compatibility constraints in the definition of a competitive equilibrium, which is not straightforward; (iii) random contracts of this type are not observed empirically, perhaps because they are not renegotiation proof: after the contract is signed, a Pareto improvement could be achieved by increasing the probability of investment to one.

By comparing (2.14) with the investment condition under full information, it emerges that adverse selection brings about an *Information premium*. It is a measure of the degree of cross-subsidization from the marginal type, whose productivity is the highest among the investing entrepreneurs, to the average investing type. Importantly, the *Information premium* need not be monotonically increasing in $\hat{\theta}$. This gives rise to the possibility of multiple steady states. In the following section, we characterize the steady states and we provide conditions for such multiplicity to arise.

Steady state characterization

Let's define a function $H(\theta)$, which describes the information premium as a function of the threshold θ:

$$H(\theta) \equiv \frac{\bar{p}(\theta)}{E\left[\bar{p}(s)|s \leq \theta\right]}, \tag{2.15}$$

and define $\bar{\theta} \equiv \arg\max_{\theta\in[0,1]} H(\theta)$, $\underline{\theta} \equiv \arg\min_{\theta\in[0,1]} H(\theta)$, $\pi(k) \equiv (1-\alpha)k^{\alpha}$, where k is the steady state level of capital which coincides with the steady state under full information. We know that $\bar{\theta}$ and $\underline{\theta}$ are well defined since $H(\theta)$ is a continuous functions and Θ is compact. The following proposition characterizes the steady states of this economy.

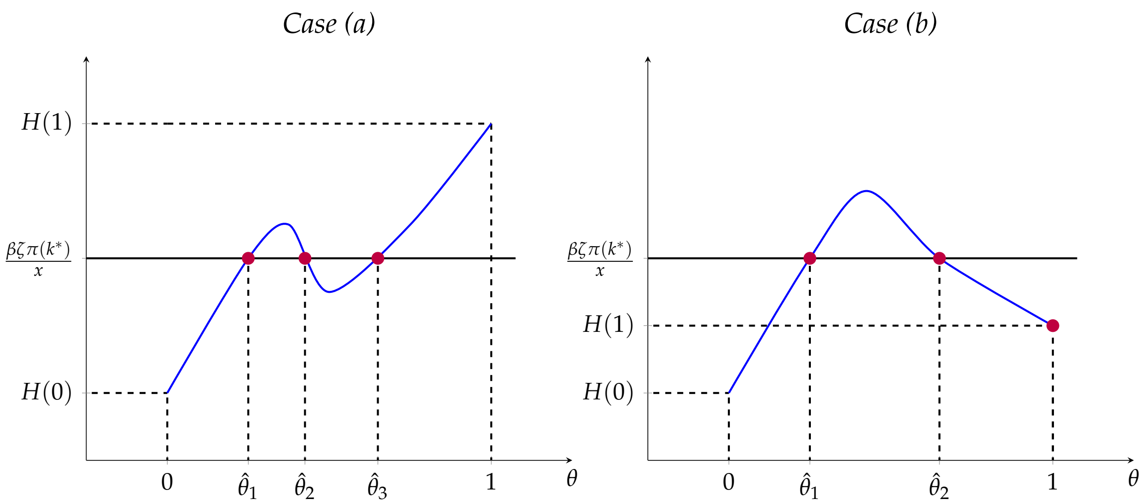
Proposition 12. *A steady state exists and:*

1. *If* $\frac{\beta\zeta\pi(k)}{x} > H(\bar{\theta})$*, at the unique steady state every type invests (*$\hat{\theta} = 1$*);*
2. *If* $\frac{\beta\zeta\pi(k)}{x} \in [H(\underline{\theta}), H(\bar{\theta})]$*, we have two cases:*
   (a) *If* $\frac{\beta\zeta\pi(k)}{x} < H(1)$*, every steady state features underinvestment (*$\hat{\theta} < 1$*);*
   (b) *If* $\frac{\beta\zeta\pi(k)}{x} \geq H(1)$*, there exist (generically) at least three steady states, one of which with full investment.*
3. *If* $\frac{\beta\zeta\pi(k)}{x} < H(\underline{\theta})$*, there exists a unique steady state with no investment.*

Proof. **Case (1):** Existence: an equilibrium with $\hat{\theta} = 1$ exists because $\frac{\beta\zeta\pi(k)}{x} > H(1)$. Uniqueness follows from $\frac{\beta\zeta\pi(k)}{x} > H(\hat{\theta})$, for every $\hat{\theta} \in [0,1]$. **Case**

(2.a): It is obvious that we cannot have an equilibrium with $\hat{\theta} = 1$ because $\frac{\beta\zeta\pi(k)}{x} < H(1)$. It remains to show that an equilibrium with $\hat{\theta} < 1$ exists. We know that $H(\underline{\theta}) \leq \frac{\beta\zeta\pi(k)}{x} < H(\bar{\theta})$. With regards to the function $H(\hat{\theta})$, observe that: (i) it is continuous in $\hat{\theta}$, and (ii) a steady state is characterized by a threshold $\hat{\theta}$ such that $H(\hat{\theta}) = \frac{\beta\zeta\pi(k)}{x}$. Existence of a steady state follows from the intermediate value theorem. Figure 2.1, Panel (a) shows an example of such case, where there exist three interior steady states ($\hat{\theta}_1$,$\hat{\theta}_2$,$\hat{\theta}_3$). **Case (2.b):** Because $\frac{\beta\zeta\pi(k)}{x} \geq H(1)$, we know that the steady state with full investment always exists. Moreover, if $\bar{\theta} \neq 1$ there will be at least another steady state with less than full investment. Generically, there will be an even number of steady states with less than full investment (so, there is an odd number of steady states in total). Figure 2.1, Panel (b) shows an example of such case, where there exist three steady states ($\hat{\theta}_1$,$\hat{\theta}_2$,1). **Case (3):** We have $H(\underline{\theta}) \geq 1$,

FIGURE 2.1: The steady states in Proposition (12), Case 2



because $H(0) = 1$. Since under full information the cost of investment for type θ is equal to one, the result follows.

□

Comparative statics

Before moving to the dynamic characterization of our economy in section (2.4), it is helpful to perform some comparative statics. We are interested in two dimensions: the fraction of *transparent* entrepreneurs λ and the distribution of types $F(\theta)$. The latter is a measure of the degree of information asymmetries in the economy, the former, conditional on a given function $\underline{p}(\theta)$, is a measure of the production possibilities.

Information asymmetries The fraction of 'transparent' entrepreneurs λ measures the level of asymmetric information, with $\lambda = 1$ being the full information case. However, since the distribution of types inside each of the two groups is independent of λ, it does not directly affect the information premium, hence the degree of adverse selection inside the pool of *opaque* entrepreneurs. It follows that at the steady state, $\hat{\theta}$ is independent of λ. Under assumption (2), it is straightforward to see that output is linear in λ:

$$Y = \left(\zeta[\lambda + (1-\lambda)F(\hat{\theta})] + \int_0^1 \underline{p}(\theta)f(\theta)d\theta\right)k^\alpha,$$

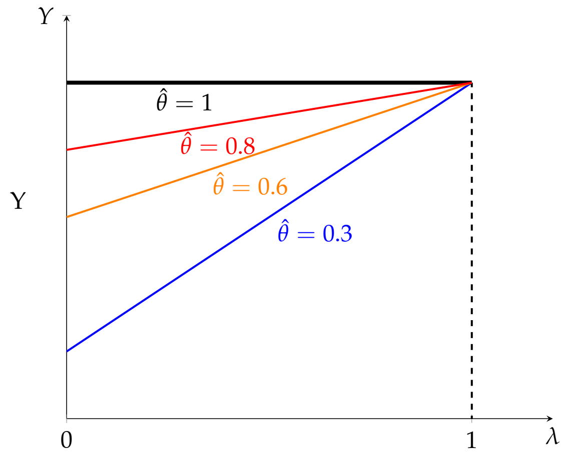
with derivative

$$\frac{\partial Y}{\partial \lambda} = \zeta(1 - F(\hat{\theta}))k^\alpha. \tag{2.16}$$

The term $\zeta(1 - F(\hat{\theta}))$ has to be interpreted as the productivity gain from shifting a mass of agents from the *opaque* to the *transparent* pool. An important feature of equation (2.16) is that the impact of a shift in λ on Y is stronger the lower the $\hat{\theta}$, that is, the higher the degree of underlying adverse selection in the economy as is it shown in Figure 2.

It is worth anticipating that $\hat{\theta}$ will generally be a function of λ when we consider a stochastic version of this economy, in the subsequent sections.

FIGURE 2.2: Comparative statics: ss. Y as a function of λ, conditional on different $\hat{\theta}$



Distribution of types Consider two distributions F, G and suppose that F first orderly stochastically dominates G, that is $F(x) < G(x)\ \forall x \in (0,1)$. Since wlog we have assumed that $\bar{p}(\theta)$ is monotonically increasing in θ, then it must be

$$\frac{\int_0^{\hat{\theta}} \bar{p}(\theta) f(\theta) d\theta}{\bar{p}(\hat{\theta}) F(\hat{\theta})} \geq \frac{\int_0^{\hat{\theta}} \bar{p}(\theta) g(\theta) d\theta}{\bar{p}(\hat{\theta}) G(\hat{\theta})}.$$

The information premium associated to the distribution F is lower than the information premium associated to the distribution G. It follows from equivalence (2.14) that in equilibrium the $\hat{\theta}$ associated with distribution F is higher. A shock that tilts the distribution of types to the right (left) can be thought as a positive (negative) productivity shock. This exercise tells us that in presence of adverse selection, the response of output will be amplified because of the general equilibrium effect that decreases (increases) the number of firms subject to adverse selection.

2.3 Constrained inefficiency

We have shown that adverse selection may lead to steady states that are inefficient with respect to the first best. However, can something be done about it?

Focusing on the first best could be a misleading benchmark, as steady states with less than full investment may still be *constrained optimal*. To answer the efficiency question one has to study whether there exists a government policy – i.e., a set of taxes and subsidies – such that: (i) it does not require more information than that available to market participants; (ii) it balances the government's budget; and (iii) it makes every agent weakly better off, and some agent strictly better off. We call an intervention *feasible* if it satisfies (i) and (ii).

Let us start with part (ii). We first assume that the government uses only two instruments: a lump sum tax $\tau > 0$ on the household sector, and a subsidy to 'opaque' firms of size ϕx, for $\phi \in [0,1]$. As it will be clear from our results, restricting attention to these instruments is without loss of generality. Because of the informational constraint (i) mentioned above, feasible subsidies cannot be targeted only at those high types who would not invest absent a government intervention (i.e., types $\theta > \hat{\theta}$). As in Philippon and Skreta (2012), all types choose to participate in any government scheme which involves investment subsidies, and it is impossible for the government (as it was for the market) to screen them. As a result, the subsidy ϕx can only be contingent on the decision to invest, and the balanced budget condition reads:

$$\tau = \phi x F(\hat{\theta}(\phi))(1 - \lambda) \tag{2.17}$$

where $\hat{\theta}(\phi) \geq \hat{\theta}$ is the investment threshold after the government's intervention.

As for point (iii), in the analysis we distinguish two cases: $\chi = 1$ and $\chi < 1$. The case $\chi = 1$ corresponds to an economy where there is no conflict of interest between households and entrepreneurs as the entrepreneurs do not get any utility from direct consumption and give all the profits back to the household. Hence, the government is not constrained by distributional concerns. The case $\chi < 1$ instead corresponds to an economy where households and entrepreneurs may have diverging interests as the households receive only a fraction χ of the profits, while entrepreneurs consume the remaining fraction

$1 - \chi$.

2.3.1 No distributional concerns $\chi = 1$

In this section we show that if all the profits are given back to the household ($\chi = 1$), a strong result applies: the government can always achieve the first best and support a steady state with full investment. To see this, first notice that under $\chi = 1$ the household budget constraint evaluated at the steady state boils down to the aggregate resource constraint

$$C = Z^{1-\alpha}K^{\alpha} - \delta K - [\lambda + (1-\lambda)F(\hat{\theta})]x.$$

This implies that: *i)* ϕ does not directly affect steady state consumption C, *ii)* $\frac{\partial C}{\partial \hat{\theta}} = (\zeta(1-\alpha)k^{\alpha} - x)(1-\lambda)f(\hat{\theta})$, hence, if investment has net positive present value, consumption is monotonically increasing in $\hat{\theta}$. It follows that under assumption (2) every intervention that increases the number of investing entrepreneurs represents a Pareto improvement. Moreover, the first best is achieved at full investment $\hat{\theta} = 1$. It is then easy to show the following proposition.

Proposition 13. *If $\chi = 1$, every steady state such that $\hat{\theta} < 1$ is constrained suboptimal. In particular, there always exists a policy ϕ that supports a steady state with $\hat{\theta} = 1$.*

Proof. At full investment, the information premium is $H(1) = \frac{p(1)}{E[p(\theta)|\theta \leq 1]}$. Moreover, since we are in steady state, it must be that $R = \beta^{-1}$. The marginal entrepreneur is $\hat{\theta} = 1$ invests if and only if

$$H(1)x(1-\phi) \leq \beta\zeta(1-\alpha)k^{\alpha}.$$

Rearranging, we get:

$$\phi \geq \frac{H(1)x - \beta\zeta(1-\alpha)k^{\alpha}}{H(1)x}.$$

Full investment is efficient whenever $x \leq \beta\zeta(1-\alpha)k^{\alpha}$. Plugging inside yields: $1 \geq \phi \geq 1 - \frac{1}{H(1)}$, which always holds since $H(1) \geq 1$. Moreover, an equilibrium with $\hat{\theta} < 1$ exists only if $xH(\bar{\theta}) \geq \beta\zeta(1-\alpha)k^{\alpha}$, which implies that $\phi \geq \frac{\beta\zeta(1-\alpha)k^{\alpha}}{xH(1)}\left(\frac{H(1)}{H(\bar{\theta})} - 1\right) \leq 0$, where the latter inequality follows from the definition of $H(\bar{\theta})$, which is the maximum $H(\theta)$ over $\theta \in [0,1]$. □

The result holds because when $\chi = 1$ the households will fully benefit from every increase in production. An interesting question is whether a Pareto improving intervention can be devised even in presence of distributional concerns. We explore this question in the next section.

2.3.2 Distributional concerns $\chi < 1$

If $\chi < 1$ part of the intervention represents a private rent to the entrepreneurs, hence the household might not necessarily benefit from full investment. To discuss under which conditions an intervention might be warranted, it is useful to start from considering how entrepreneurs are affected by the policy. This is summarized by the following lemma.

Lemma 5. *The participation constraint for the entrepreneurs is never binding. Every entrepreneur is at least as well off from an intervention $\phi > 0$. Moreover, the utility of entrepreneurs as a group is monotonically increasing in ϕ.*

Proof. Crucial to the result is the fact that the steady state level of capital per productive firm k is pin down by the parameters of the economy and is neither affected by ϕ, nor by $\hat{\theta}$. Hence, entrepreneurs' revenue at the steady state is not affected by the intervention. It follows that 'transparent' entrepreneurs that do not receive the subsidy are as well off as compared to the *laissez faire*. So are the θ-type 'opaque' entrepreneurs that do not invest under the policy as well as the marginal type (i.e. the set of θ-type entrepreneurs such that $\theta \geq \hat{\theta}(\phi)$). It is immediate to see that, by decreasing the pooling rate, the subsidy will make the set of θ-type 'opaque' entrepreneurs such that $\theta < \hat{\theta}(\phi)$ strictly better off. Since $\hat{\theta}(\phi)$ is monotonically increasing in ϕ, by increasing the number of investing types, a higher subsidy ϕ increases the utility of the entrepreneurs as a group. $\square$

This result tells us that to characterize the set of Pareto improving interventions, it is sufficient to look at the set of policies that make the household better off as compared to the *laissez faire*. Let $C(\phi)$ be the steady state consumption of the representative household as a function of the policy ϕ. From the household budget constraint, this can be expressed as

$$\begin{aligned} C(\phi) = (\alpha + \chi(1-\alpha))Z^{1-\alpha}K^{\alpha} - \delta K - [\lambda + (1-\lambda)F(\hat{\theta}(\phi))]x \\ + \left(\frac{1-\chi}{\beta}\right) \underbrace{[\lambda + (1-\lambda)(1-\phi)F(\hat{\theta}(\phi))]}_{\equiv B(\phi)} x. \end{aligned} \tag{2.18}$$

We can define the Pareto set as follows.

Definition 6. Pareto set:

$$\Phi \equiv \{\phi \mid C(\phi) > C(0)\}.$$

Let ϕ_E be the policy that maximizes the utility of the entrepreneurs, then, by definition (6) together with lemma (5), it must be:

Lemma 6. $\phi_E = \sup\{\Phi\}$.

We can now characterize the Pareto set. From (2.18), notice that ϕ enters directly the equation from consumption through the term $B(\phi)$. However, this term is non-linear and absent more restrictive assumptions on the distribution of types $F(\theta)$, the derivative $\frac{\partial B(\phi)}{\partial \phi}$ (the derivation is shown in appendix), cannot be signed. Not being able to establish global regularity conditions on the function $C(\phi)$ prevents us from deriving global conditions for a Pareto improving intervention. However, we are still able to establish a local condition around the *laissez faire* equilibrium. The following proposition answers the question whether there exists a marginal intervention that can improve upon the market equilibrium.

Proposition 14. *Let* $\frac{\beta\zeta\pi(k)}{x} < H(1)$*. A local Pareto improving government intervention exists if and only if:*

$$\underbrace{-\frac{\partial}{\partial\hat{\theta}(\phi)}\left(\frac{\mathbb{E}[\bar{p}(\theta)|\theta \leq \hat{\theta}(\phi)]}{\bar{p}(\hat{\theta}(\phi))}\right)}_{\text{increase in mispricing for the marginal type}} \leq \underbrace{\frac{f(\hat{\theta})}{F(\hat{\theta})}\left[\frac{(1-\beta)\left[x+\zeta k\right]+\chi(\beta\zeta(1-\alpha)k^{\alpha}-x)}{(1-\chi)\beta\zeta\pi(k)}\right]}_{\text{Household's gain from investment}}$$

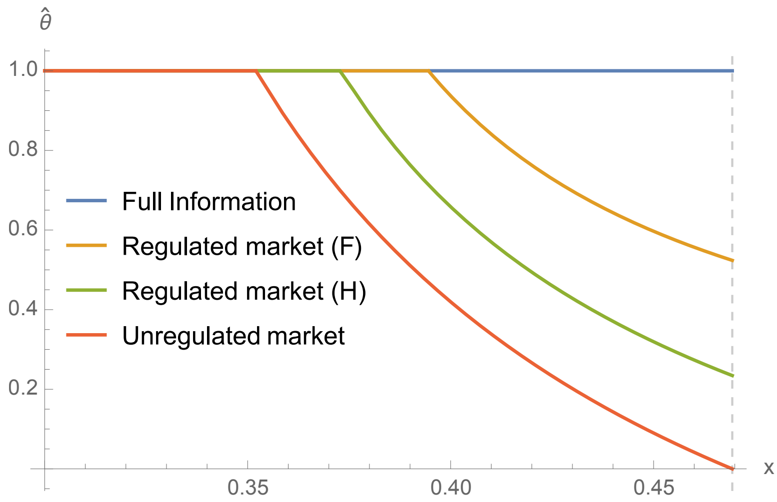
where $\hat{\theta}$ *is the 'best' steady state for the household, absent government interventions.*

Proof. The expression follows from the first derivative of equation 2.14 evaluated at no intervention (i.e., $\phi = 0$). To derive it, simply observe that: (i) when $\phi = 0$ we have $\left(1 - \frac{\beta\zeta\pi(k)}{x}\frac{\mathbb{E}[\bar{p}(\theta)|\theta\leq\hat{\theta}(\phi=0)]}{\bar{p}(\hat{\theta}(\phi=0))}\right) = 0$; (ii) $\frac{\beta\zeta\pi(k)}{x} > 0$, and (iii) because of Assumption 2 we also know that $F(\hat{\theta}) > 0$. □

Proposition (14) identifies a sufficient condition for constrained suboptimality of the equilibria of our economy. The 'best' competitive equilibrium allocation is locally suboptimal if the rise in mispricing is bounded above, and it is more than compensated by the respective increase in the aggregate investment. Intuitively, the government raises funds from the households, through the tax τ. The household sector is better off after a government's intervention only if the increase in its proceeds from capital lease and additional loans, more than offset the tax. In turns, the increase in profits from capital leases are proportional to the rise in aggregate demand for capital by the entrepreneurial sector. The condition in (14) states that the minimal subsidy which induces a marginally higher fraction of types to invet needs to be small enough, relative to the corresponding increase in the fraction of investment types.

In Figure 2.3 we provide an example of constrained suboptimality. This example uses the calibration in section (2.4) and plot the threshold $\hat{\theta}$ as a function of the cost of investment x in four scenarios: (i) in the full information benchmark; (ii) at the laissez faire equilibrium (Unregulated market); (iii) at the optimal subsidy for the household ϕ_H (Regulated Market, H); and (iv) at the optimal subsidy for the entrepreneurs ϕ_E (Regulated Market, F).

FIGURE 2.3: An example of constrained suboptimality



Note: the figure obtains for the parametrization in table (2.1)

2.4 Informational shock

So far, we characterized analytically the steady states of the non-stochastic version of our economy; we provided conditions for the steady states to be constrained suboptimal; and we discussed how a government can set-up a balanced budget policy to implement a Pareto improvement. Such policy involve consumption taxes on households, and investment subsidies to firms subject to informational asymmetries.

We now want to look at the dynamics of the economy in response to the *informational shocks* we introduced in equation (2.9). Following standard practices, we study the local dynamics around the 'best' deterministic steady state of our economy and we draw impulse responses to a negative shock that decreases λ, that is, increases the pool of firms subject to asymmetric information.

2.4.1 Calibration

To solve the model, we need to make assumptions on the functional form of the distribution of types $F(\theta)$ as well as the entrepreneurs' probability of being productive $\underline{p}(\theta)$. We assume that *i)* types are uniformly distributed, i.e. $F(\theta) \sim$

TABLE 2.1: Parameters used in the calibration

Parameter	Value	Target
α	0.36	
β	0.99	
γ	2	
δ	0.025	
λ	1	
x/Y	0.122	$\hat{\theta} = 0.7$
ζ	0.2	
a		
ρ_θ	0.8	
ρ	0.9	
χ	1	

$U[0,1]$, *ii)* $\underline{p}(\theta)$ is affine in θ, i.e. $\underline{p}(\theta) = a + \rho_\theta \theta$. Moreover we assume that the household utility is a CRRA of the form $u(c) = c^{1-\gamma}/(1-\gamma)$. A summary of the parameters are presented in table (2.1).

The calibration is quarterly. The values for α, δ, β and γ are standard in the literature. We assume that at the steady state there is full information, $\lambda = 1$, and the economy operates at full investment. Investment increases the probability of being productive ζ by 20 percent. However, we assume that 30 percent of the entrepreneurs would not invest if subject of asymmetric information ($\hat{\theta} = 0.7$). To target this value, the cost of investment x is set to about 12 percent of GDP.

2.4.2 Impulse response functions

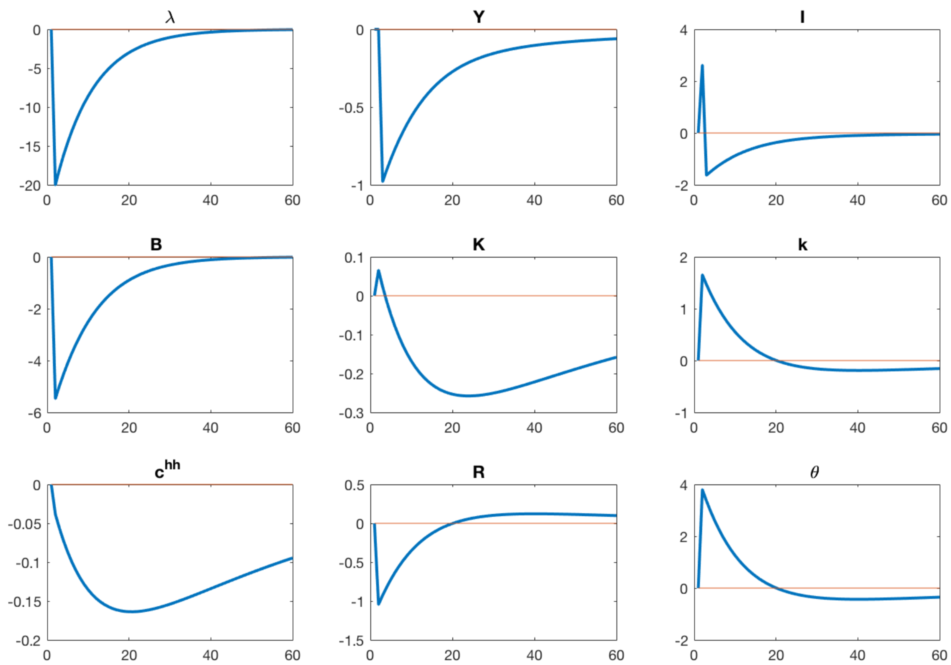
We consider a one time negative shock to λ that affects 20 percent of the firms. We solve the model using the perfect foresight simulation algorithm implemented in Dynare which uses the Fair and Taylor (1983) method. This method imposes certainty equivalence on the nonlinear model, as such, it traces out the implications of not linearizing the equilibrium equations without shock uncertainty. We therefore compute impulse responses under perfect foresight starting from the non-stochastic steady state. The results are presented in figure (2.4).

A negative shock to λ decreases entrepreneurs' demand for credit: B falls on impact of about 5 percent. In response to the decrease in investment opportunities, households relocate savings toward investment in physical capital I that increases of about 2 percent. The overshooting of investment in capital despite the decline in its marginal productivity - the interest rate R falls about 1 percent - might seem counterintuitive at first. However, it is explained by two facts: *i)* output Y does not fall on impact as the response of output is lagged by one period, *ii)* expectations of lower output in the future increase precautionary savings. Consumption decreases marginally and the excess of savings is redirected toward the unique alternative investment opportunity, i.e. physical capital. Interestingly, the model is able to generate a dynamic which resembles a fly-to-quality. However, differently from the supply side literature where a fly-to-quality originates from constraints in the supply of credit, in our model the driver is a contraction in the demand generated by the emergence of an information premium.

As output contracts, investment in physical capital declines and remains subdue for a prolonged period of time. The decline in the number of productive entrepreneurs causes the capital per productive entrepreneur to increase and the marginal product of capital to decline. In turn this provides incentive to reduce the capital stock. On the other hand, after the initial drop, B picks us over time as λ reverts back to its steady state value, helping to restore the productivity of capital. The net effect is that the capital stock declines for 20 quarters before slowly recovering. The slow recovery of investment following the great recession has often been advocated as a 'puzzle' as traditional RBC models would predict a fast recovery in both investment and the capital stock. Notably, in our setting an informational shock is able to generate a very slow recovery of capital and investment even absent any adjustment cost.

A last question concerns how the degree of adverse selection inside the

FIGURE 2.4: Impulse Responses to an informational shock



Note: The figure shows impulse-responses to a one time shock to λ solved using Dynare perfect foresight algorithm based on Fair and Taylor (1983). Results are presented in percentage deviation from the steady state.

pool of 'opaque entrepreneurs', summarized by $\hat{\theta}$, evolves over time. Looking at the last two quadrant in figure 2.4, it is clear that the dynamic of $\hat{\theta}$ mirrors movements in the interest rate. The decline in R acts as a countervailing force to the decline in λ, mitigating the adverse selection problem. On impact $\hat{\theta}$ rises of about 4 percent with respect to the steady state, to then decline over time.

2.5 Conclusions

In this paper, we study a stylized Real Business Cycle model in which a fraction of entrepreneurs are *opaque* in that subject to informational asymmetries. When adverse selection is not severe, competitive allocations are efficient and the economy operates at full investment; however, if informational asymmetries are sufficiently strong, competitive equilibria feature underinvestment. We show that a balance-budget policy which does not require more information than the information market participants have and that subsidizes investment and taxes consumption can improve upon the market allocation at the steady state. Absent intervention, the resulting competitive equilibria are therefore constrained suboptimal.

We then use our framework to study the effect of purely informational shocks, interpreted as shocks that shift the proportion of entrepreneurs subject to asymmetric information. We show that, in presence of adverse selection, these shocks may be an important driver of business cycle fluctuations. In particular, they are able to generate prolonged periods of low investment and low growth, despite the absence of any fundamental shock to the production possibilities of the economy.

Chapter 3

Dynamic Fiscal Limits and Fiscal-Monetary Interactions

3.1 Introduction

In recent years monetary policy has played a prominent role in the determination of fiscal sustainability in advanced countries. Measures of monetary easing adopted by central banks aimed at sustaining inflation and economic activity have depressed interest rates and thus helped fiscal authorities service their debt. Yet, the zero lower bound (ZLB) has constrained the role of monetary policy as a counter-cyclical tool, calling for an active role of fiscal policy in business cycle stabilization. Despite near zero interest rates, expansionary fiscal policies and fiscal policy shocks due to financial sector bailouts, in some cases have contributed to increases in risk premia on government debt and undermine market confidence. Looking forward, the perspective of a normalization of interest rates, raises further concerns on future debt sustainability in many advanced economies, e.g. Beck and Wieland (2017).

In light of these consideration, the impact of monetary policy on the sustainability of public debt and, in turn, the ensuing implications for the transmission of public spending shocks are of particular relevance in the current policy debate. In this paper, we focus on three main questions:

1. How does the degree of monetary policy activeness affect fiscal sustainability?

2. How is fiscal sustainability affected by persistent periods at the ZLB?

3. How does the response of monetary policy affect the sustainability of spending shocks?

This paper explores these issues through the lens of a dynamic stochastic general equilibrium model (DSGE) based on the seminal work of Bi (2012) . Fiscal sustainability is interpreted as the probability that the country will be *able* to service its debt in the future. Central to this class of models is the concept of *fiscal limit*, which is obtained by simulating the present discounted value of all maximum future primary surpluses conditional on the initial state of the economy.

Compared to the existing literature, we introduce two main innovations. First, differently from Bi, Leeper, and Leith (2013) , who, in computing the fiscal limit, assume that the monetary authority is always able to peg inflation to the target, we assume that the monetary authority follows a Taylor rule. Inflation dynamics then affect the fiscal limit distributions that become endogenous to monetary policy. Second, we introduce a consumption preference shock as in Erceg and Lindé (2014), in order to mimic the economic developments that made the ZLB binding during the recent crisis. On the one hand, the introduction of a demand-side shock allows us to analyze how changes in the consumption/saving behavior of households affect the government's capacity to service its debt at and away from the ZLB. On the other hand, our framework make it possible to study the consequences on fiscal sustainability of a fiscal policy shock when monetary policy is constrained by the ZLB and government debt is risky.

Our results indicate that the responsiveness of monetary policy to changes in inflation can considerably affect fiscal sustainability. A looser monetary policy stance – i.e., a lower coefficient in the Taylor rule – implies larger fluctuations of inflation away from its target in response to exogenous shocks. In turn, this leads to higher price adjustment costs, lower output and a smaller tax base, which negatively affect fiscal sustainability.

In addition, our analysis suggests that to assess the effect on debt sustainability of aggregate demand shocks is important to distinguish the two channels at play. An aggregate demand shock, by lowering output, decreases the tax revenue (growth channel), but also depresses interest rates reducing interest repayments (interest rate channel). These two channels move in opposite direction. A binding ZLB, by preventing monetary accommodation, amplifies the growth channel, while muting the interest rate channel with detrimental consequences on fiscal sustainability. Indeed, while the government has the possibility to increase the tax rate, using it as an imperfect substitute for monetary policy in order to support inflation, this is costly in terms of output and the resulting tax revenue is depressed.

Finally, we find that making the debt limit endogenous to monetary policy and ZLB considerations is important to better understand the transmission of spending shocks. In normal times, a looser monetary policy stance reduces the margin for expansionary spending shocks, which, by increasing spreads, after the initial expansion may induce a recessionary phase as distortionary taxation has to increase to stabilize debt. In contrast, we find that the positive macroeconomic effects of public spending shocks are larger at the ZLB when monetary policy is constrained and a government spending shock may actually improve debt sustainability. However, the effectiveness of the spending shock depends on the underlying monetary policy stance. A more responsive monetary policy is better able to peg inflation expectations to the target. In turns this dampens the deflationary pressure during periods of binding ZLB, allowing the real interest rate to fall further down, this way sustaining aggregate demand and increasing the fiscal space for the government.

Our analysis is linked to the literature studying the implications of the fiscal theory of the price level on inflation determination, and the different combination of active/passive fiscal and monetary policy (see Davig, Leeper, and Walker (2011) and Leeper and Leith (2016) among others). Differently from them, however, we study how *active* monetary policy regimes or temporary suspension of monetary policy rules (i.e. periods of binding ZLB) affect debt limits.

In the spirit of Bi (2012), we model sovereign default as a random event, whose likelihood increases with the level of debt. Hence, we do not propose a theory for sovereign default, as in Uribe (2006), where the focus is on the equilibrium behaviour of default rates and sovereign risk premia. Rather, our objective is to analyse how the maximum amount of debt that a country is able to tolerate changes depending on the type of monetary policy followed and by the fiscal and macroeconomic fundamentals. Moreover, the paper focuses on the *capacity* to pay, rather than the *willingness* to pay, instead a crucial element of the analysis of Arellano (2008). We also abstract from considerations of self-fulfilling dynamics and multiple equilibria as in Lorenzoni and Werning (2013) or on the capacity of monetary policy to prevent them as in Corsetti and Dedola (2016).

The rest of the paper is structured as follows. Section 3.2 outlines the general equilibrium model. Section 3.3 presents the methodology for its numerical solution alongside the calibration parameters. Section 3.4 assesses the role of monetary policy and the ZLB in the determination of the debt limit. Section 3.5 introduces our forward-looking fiscal indicator. In section 3.6, finally, we apply our fiscal indicator to evaluate the role of spending shocks in a set of policy scenarios, including the analysis of periods of binding ZLB. Section 3.7 concludes.

3.2 The model

The model builds on the works by Bi (2012) and Bi, Leeper, and Leith (2013) by introducing preference shocks *á la* Erceg and Lindé (2014) and allowing for the presence of a ZLB on the nominal risk-free interest rate. Time is discrete and denoted as $t = 0, 1, 2, ..., \infty$. The closed economy is populated by a representative household, who consumes, works, owns monopolistically competitive firms producing differentiated intermediate goods and perfectly competitive firms producing a homogeneous final good, and invests in two types of state-noncontingent assets, namely risk-free bonds and risky (i.e., defaultable) government bonds.

3.2.1 The representative household

The representative household maximizes the following initial utility function:

$$\max_{\{C_t, N_t, B_t, B_t^F\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{(C_t - C\nu_t)^{1-\gamma}}{1-\gamma} - \frac{\chi_0}{1+\frac{1}{\chi}} N_t^{1+\frac{1}{\chi}} \right], \tag{3.1}$$

subject to the flow budget constraint:

$$C_t + \frac{B_t}{R_t} + \frac{B_t^F}{R_t^F} = (1-\tau_t)(W_t N_t + \Upsilon_t) + Z_t + \frac{B_{t-1}^d}{\Pi_t} + \frac{B_{t-1}^F}{\Pi_t}, \tag{3.2}$$

where E_0 denotes the expectations operator, β the household's discount factor, γ its relative risk aversion and χ its Frisch elasticity. Moreover, C_t denotes private consumption, N_t hours of labour, τ_t the tax rate on wage income and profits, W_t the (real) wage rate, Υ_t the representative firm's profits, Π_t (gross) inflation, Z_t transfers from the government to the households, B_t risky (i.e. defaultable) government bonds, with associated (gross) interest rate R_t, and B_t^F risk-free bonds, with associated (gross) interest rate R_t^F, at time t. Notice that $B_{t-1}^d \equiv (1-\Delta_t)B_{t-1}$ denotes the part of real outstanding debt actually repaid and Δ_t is the haircut on outstanding debt in case of default. As in Erceg and Lindé (2014), the utility function depends on the household's current consumption C_t as deviation from a reference level $C\nu_t$. The exogenous consumption taste shock ν_t lowers the reference level and marginal utility of consumption and follows an $AR(1)$ process

$$\nu_t = (1-\rho_v)\nu + \rho_\nu \nu_{t-1} + \sigma_\nu \epsilon_t, \qquad \epsilon_t \sim N(0,1). \tag{3.3}$$

The first order conditions of the household problem define the following labor supply schedule

$$N_t = \left[\frac{(1-\tau_t) W_t}{\chi_0 \left(C_t - C\nu_t\right)^{\gamma}} \right]^{\chi}, \tag{3.4}$$

and the standard consumption Euler equation for riskless bonds

$$\frac{1}{R_t^F} = \beta E_t \left[\left(\frac{C_{t+1} - C\nu_{t+1}}{C_t - C\nu_t} \right)^{-\gamma} \frac{1}{\Pi_{t+1}} \right]. \tag{3.5}$$

The Euler equation for the (risky) government bonds reads

$$\frac{1}{R_t} = \beta E_t \left[(1 - \Delta_{t+1}) \left(\frac{C_{t+1} - C\nu_{t+1}}{C_t - C\nu_t} \right)^{-\gamma} \frac{1}{\Pi_{t+1}} \right]. \tag{3.6}$$

where

$$\Delta_t = \begin{cases} 0 & \text{if } B_{t-1} < B_t^* \\ \delta & \text{if } B_{t-1} \geqslant B_t^*, \end{cases} \tag{3.7}$$

where δ is the size of the haircut if default. As in Bi (2012), the default scheme depends on the effective fiscal limit B_t^*. Each period B_t^* is drawn stochastically from the distribution of fiscal limits conditional on the state of the economy at time t, as explained in section (3.2.6).

3.2.2 Final goods production

The single final output good Y_t is produced using a continuum of differentiated intermediate goods $Y_t(i)$. Competitive final good firms buy the differentiated goods produced by intermediate goods producers and combine them according to an aggregate function which has the CES (constant elasticity of substitution) form

$$Y_t = \left[\int_0^1 Y_t(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}. \tag{3.8}$$

Cost minimization for final good producers results in the demand curve for the generic intermediate good i

$$Y_t(i) = \left[\frac{P_t(i)}{P_t} \right]^{-\theta} Y_t \tag{3.9}$$

and an associated price index for the final good

$$P_t = \left[\int_0^1 P_t(i)^{1-\theta} di \right]^{\frac{1}{1-\theta}}. \tag{3.10}$$

3.2.3 Intermediate goods production

A continuum of intermediate goods $Y_t(i)$ for $i \in [0,1]$ is produced by monopolistically competitive firms, each of which produces a single differentiated good. Intermediate goods firms are subject to Rotemberg adjustment costs that penalise large price changes in excess of steady-state inflation rates. Producer i's maximisation problem reads

$$\max_{P_t(i)} E_0 \sum_{t=0}^{\infty} R_{0,t}^F \left[P_t(i) Y_t(i) - mc_t P_t Y_t(i) - \frac{\phi}{2} \left(\frac{P_t(i)}{P_{t-1}(i)\Pi} - 1 \right)^2 P_t Y_t \right] \tag{3.11}$$

$$s.t. \quad Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\theta} Y_t, \tag{3.12}$$

where $R_{0,t}^F \equiv \beta^t \left[\frac{C_t - C\nu_t}{C_0 - C\nu_0} \right]^{-\gamma}$, is the household discount factor, $P_t(i)$ is the price chosen by firm i and P_t is the nominal aggregated price level. Intermediate good producers are endowed with a linear production function

$$Y_t(i) = A_t N_t(i) \tag{3.13}$$

where A_t is total factor productivity which follows an exogenous AR(1) process of the form

$$\ln\ A_t = (1-\rho_v)\ln\ A + \rho\ _A \ln\ A_{t-1} + \sigma_A \epsilon_t, \qquad \epsilon_t \sim N(0,1) \tag{3.14}$$

which, in equilibrium, implies the real marginal cost $mc_t = W_t / A_t$.
In a symmetric equilibrium, the first-order condition gives the non-linear New Keynesian Phillips curve under Rotemberg pricing

$$(1-\theta) + \theta mc_t - \phi \frac{\Pi_t}{\Pi}\left(\frac{\Pi_t}{\Pi} - 1\right) + \phi\beta E_t\left[\left(\frac{C_{t+1} - Cv_{t+1}}{C_t - Cv_t}\right)^{-\gamma} \frac{Y_{t+1}}{Y_t}\frac{\Pi_{t+1}}{\Pi}\left(\frac{\Pi_{t+1}}{\Pi} - 1\right)\right] = 0 \tag{3.15}$$

where ϕ parametrizes Rotemberg (quadratic) price adjustment costs and θ is the elasticity of substitution between goods. Intermediate goods producers' monopolistic real profits are:

$$\Upsilon_t = Y_t - mc_t Y_t - \frac{\phi}{2}\left(\frac{\Pi_t}{\Pi} - 1\right)^2 Y_t. \tag{3.16}$$

3.2.4 Fiscal and Monetary Policy

The government budget constraint is determined by the following equation

$$\frac{B_t}{R_t} + T_t = \frac{(1-\Delta_t)B_{t-1}}{\Pi_t} + G_t + Z_t$$

where B_t is real debt, Δ_t is the haircut in case of default, T_t is tax revenue, G_t is government consumption and Z_t are fiscal transfers to the household. Public consumption follows an exogenous AR(1) process:

$$\log G_t = (1-\rho_G)\log G + \rho_G \log G_{t-1} + \sigma_G \varepsilon_t, \quad \varepsilon_t \sim N(0,1) \tag{3.17}$$

while fiscal transfers and labor tax rate respond to debt-targeting rules:

$$Z_t = \max\{Z - \mu^z(B_t - B) + \eta_t^z, \underline{Z}\}, \tag{3.18}$$

$$\tau_t = \tau + \mu^\tau(B_t - B) \tag{3.19}$$

where B denotes the steady-state real debt level and $\underline{Z}$ indicates the minimum level of transfers politically feasible,[1] while

$$\eta_t^z = \rho^\tau \eta_{t-1}^z + \sigma^z \varepsilon^z, \quad \varepsilon_t^z \sim N(0,1) \tag{3.20}$$

Tax revenue is given by:

$$T_t = \tau_t(W_t N_t + Y_t). \tag{3.21}$$

Turning to the central bank, it is assumed to follow a (truncated) Taylor rule subject to the zero lower bound (ZLB):

$$R_t^F = \max\left\{R^F \left(\frac{\Pi_t}{\Pi}\right)^\alpha \eta^R, 1\right\} \tag{3.22}$$

where Π is the target inflation rate and η^R is a monetary policy shock

$$\eta_t^R = \rho^\tau \eta_{t-1}^R + \sigma^R \varepsilon^R, \quad \varepsilon_t^R \sim N(0,1). \tag{3.23}$$

3.2.5 Aggregate resource constraint

Closing the model economy, the aggregate resource constraint is given by

$$C_t + G_t = Y_t \left[1 - \frac{\phi}{2}\left(\frac{\Pi_t}{\Pi} - 1\right)^2\right], \tag{3.24}$$

whereby transfers Z_t cancel out as they simply redistribute resources between the household and the government.

[1] Parameter $\underline{z}$ aims at capturing the political constraints faced by the government in providing households with a minimum level of transfers.

3.2.6 Distribution of the fiscal limits

Following Bi (2012), we quantify the risk of sovereign default starting from fiscal limits that arise from the tax revenue side of the government's budget constraint in presence of distortionary taxation. At the peak of the Laffer curve, tax revenues reach their maximum and, for a given level of total government expenditures, the present value of primary surpluses is maximised. Revenues, expenditures and discount rate vary with the shocks hitting the economy, generating a distribution for the maximum debt-GDP level that can be supported.

Differently from Bi, Leeper, and Leith (2013) , who, in computing the fiscal limits, assume that the monetary authority is always able to peg inflation to the target, we assume that the monetary authority follows a Taylor rule and we allow inflation to vary around its steady state. Inflation dynamics then affect the fiscal limit distributions that become endogenous to monetary policy. That allows us to study how the fiscal limits depend on the monetary policy stance and respond to the presence of occasionally binding ZLB constraints.

Formally, the stochastic processes governing the exogenous state induce stochastic processes for both the tax rate τ_t^{max} and the associated maximum tax revenue $T_t^{\max}$. Hence, we can write

$$T_t^{\max} = T^{\max}(A_t, \nu_t, G_t, \eta_t^z, \eta_t^R)$$

where the function T^{max} maps the current state into the tax revenue at the peak of the Laffer curve. The fiscal limit is defined as the discounted sum of expected maximum primary surpluses in all future periods.

$$B^* = E \sum_{t=0}^{\infty} \beta^t \beta_p \frac{1}{R_t^F} \left[T^{\max}(A_t, \nu_t, G_t, \eta_t^z, \eta_t^R) - G_t - \underline{Z} \right] \tag{3.25}$$

where $\frac{1}{R_t^F} = \beta \left[\frac{C(A_{t+1}, \nu_{t+1}, G_{t+1}, \eta_{t+1}^z, \eta_{t+1}^R) - C\nu_{t+1}}{C(A_t, \nu_t, G_t, \eta_{t+1}^z, \eta_{t+1}^R) - C\nu_t} \right]^{-\gamma} \left(\frac{1}{\Pi(A_{t+1}, \nu_{t+1}, G_{t+1}, \eta_{t+1}^z, \eta_{t+1}^R)} \right)$.
The stochastic discount factor is obtained when tax rates are at the peak of the

Laffer curve, but modified to allow for a political risk parameter β_p. The distribution of fiscal limits are then computed using Markov Chain Monte Carlo simulations. However, while the assumption of a fixed inflation rate allows to compute debt limits independently from the model solution, letting inflation free to vary requires solving the full non-linear model first before computing Monte Carlo simulations.

3.3 Numerical solution and calibration

The model is solved in two stages. In the first stage, in order to simulate the debt limit distributions, the model is solved conditional on the government setting the tax rate at the peak of the Laffer curve. In the second stage, the model is solved conditional on the government following the tax rule 3.19. The debt limit distributions obtained in the first stage are used to compute the state contingent probability of default at each point in time according to 3.7.

The use of a nominal model presents several problems. The most prominent one is that the computation of the debt limit distributions are not anymore independent on the equilibrium conditions of the model, as they are in Bi (2012) and Bi, Leeper, and Leith (2013) . The specific problem lies in the fact that the revenue-maximising tax rate (i.e., the peak of the Laffer curve, which is crucial in the determination of the debt limit distribution) now depends non-linearly on the real wage and the inflation rate. No functional form is available to determine the equilibrium level of these three endogenous variables. Hence, the equilibrium relationships of the three variables need to be solved numerically, given specific values for the state variables and the parameters.

The introduction of a risk-free bond allows us to simplify the computation of the maximum tax rate, the crucial element in the derivation of the debt limit, while preserving the unique equilibrium relation among the endogenous variables, given the state variables and the parameters of the model. Indeed, the introduction of a risk-free bond alongside the government bond allows us to pin down the path of consumption independently of the probability of default. Hence, when we solve the model for the maximum tax rate, the real wage and the inflation rate, we need not consider government debt, with considerable benefits in terms of computational time.

Given this assumption, we limit the number of state variables to five (TFP, government consumption, discount factor shock, transfers, monetary policy shock) and the number of numerically determined control (or jump) variables to three. Notice that a "solution of the model" includes a set (i.e., matrix) of one-to-one relationships between a specific value for the vector of state variables and a specific value for the vector of control variables. So, the model needs to be solved only once before the simulation of the debt limit distribution. When we calculate the debt limit distributions all the control variables are readily available as a function of the state variables either through functional forms or through the one-to-one relationships between state and control variables established in our first step. The solution strategy is presented in Appendix B.1 and relies upon the monotone map method based on Coleman (1991) and Davig, Leeper, and Walker (2011) . The use of global solution methods allows us to deal explicitly with the non-linearities associated with the ZLB constraint.

The calibration is presented in table 3.1. The model is calibrated at quarterly frequencies. The calibration is heavy reliant on Bi, Leeper, and Leith (2013) that calibrate fiscal parameters to match average EU-14 data from 1971 to 2007. In steady state, government purchases (public consumption) are 21 percent of GDP and lump-sum transfers are 18 percent of GDP. The tax adjustment parameter (μ_τ) is calibrated to 0.5 at an annual rate, which is close to the average of estimates in EU-14, and the tax rule targets a debt to GDP ratio of 110 percent for 'high-debt countries'. We set the Frisch elasticity to 0.42 which is close to the value used by Lindé and Trabandt (2017), who use the same specification for the discount factor shock. Upon default, we assume a haircut of 10 percent as in Cruces and Trebesch (2013) . The other parameters are standard

Parameters		Value	Source
Discount factor	β	0.99	BLL (2013)
Risk aversion	γ	1	BLL (2013)
Public consumption/GDP	g/y	21%	BLL (2013)
Transfers/GDP	z/y	18%	BLL (2013)
Tax rule	μ_τ	0.5/4	BLL (2013)
Inflation	π	3% (annual)	BLL (2013)
Taylor rule	α	1.5	BLL (2013)
Debt/GDP	b/y	110% (annual)	high-debt country
Frisch elasticity	$1/\chi$	0.42	Linde and Trabandt (2017)
Haircut if default	δ	10%	Cruces and Trebesch (2013)
TFP	a	1	standard
Labor supply	n	1	standard
TFP shocks	ρ_a	0.85	standard
TFP shocks	σ_a	0.022	standard
Preference shocks	ρ_v	0.85	standard
Preference shocks	σ_v	0.022	standard
Public consumption shocks	ρ_g	0.85	standard
Public consumption shocks	σ_g	0.01	standard
MP shocks	ρ_r	0.85	standard
MP shocks	σ_r	0.01	standard

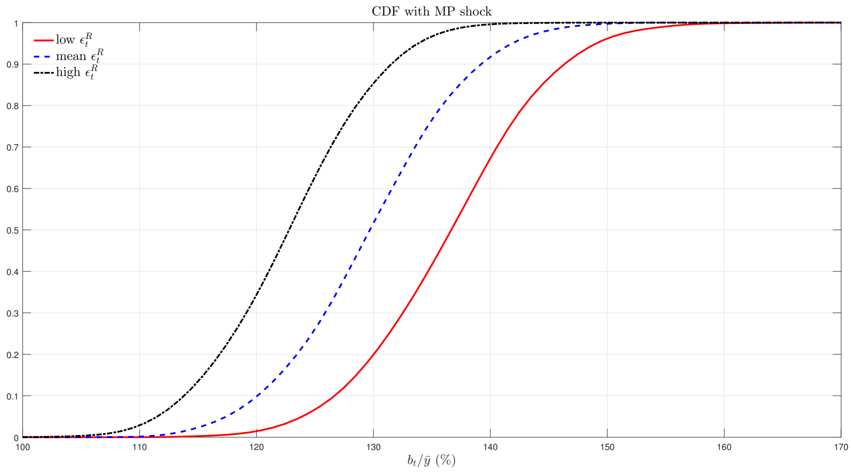
TABLE 3.1: Calibration

in the literature.

3.4 Monetary policy stance and debt limit determination

This section studies the role played by monetary policy in the determination of the debt limit distributions. Monetary policy can affect debt limits by reducing interest payments (direct interest rate channel) and by affecting the level of economic activity and, as a consequence, the path of primary balances (indirect growth channel). The analysis will first show the impact of pure monetary policy shocks, in which the interest channel plays a prominent role. To explore the role of the indirect growth channel we study different degrees of reactivity to changes in inflation in the monetary policy rule (α in eq. 3.22) and then evaluate the determination of debt limits during periods of binding zero-lower bound constraint.

FIGURE 3.1: Impact of monetary policy shocks on debt limit distributions

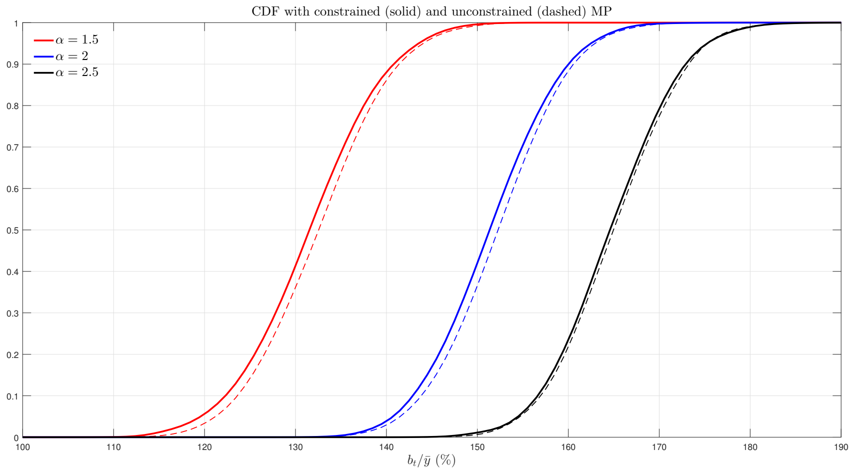


Note: Debt limit distributions after monetary policy shocks reducing (red solid line) and increasing (black dashed/dotted-line) the risk-less interest rate . The blue dashed line indicates the debt distribution when the interest rate shock ϵ_t^R is at steady state.

This analysis sheds light into an important aspect of fiscal-monetary policy interactions, often overlooked in the DSGE literature. Even in most recent studies (see, for example, Bi, Leeper, and Leith (2013)), inflation is fixed at its steady-state level when debt limit distributions are computed, so that no role is played by inflation and, thus, monetary policy in determining fiscal sustainability. In contrast, we allow inflation to drift away from its target in response to shocks in fundamentals and policy, so that we can evaluate the impact of these shocks on the sustainability of public debt under different macroeconomic conditions and policy regimes.

Figure 3.1 shows the impact on the debt limit distribution of both contractionary and expansionary monetary policy shocks as applied to the baseline calibration. The direct interest rate channel and the indirect growth channel move debt limits in the same direction. An expansionary shock is associate with a reduction in interest rates and an increase in output. This increases debt sustainability, as it reduces the probability that the debt limit will be below any given level of the debt-to-GDP ratio.

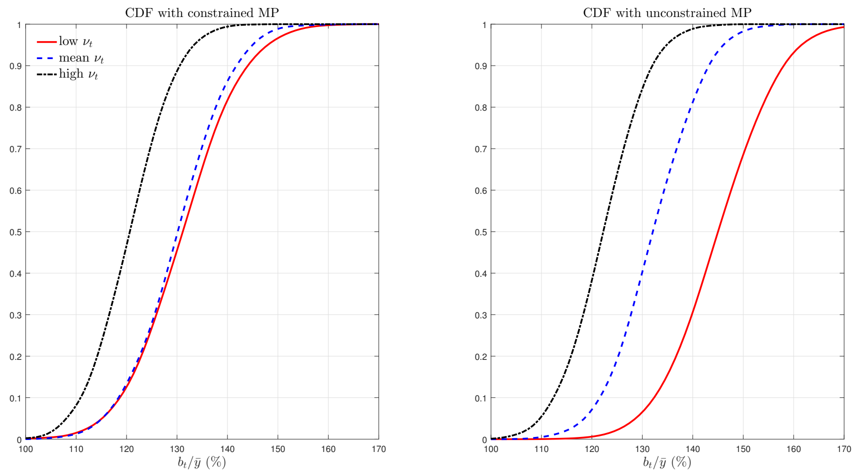
FIGURE 3.2: Impact of monetary policy shocks on debt limit distributions



Note: The figure presents how the debt limit distributions change for three different values of the inflation coefficient in the Taylor rule (1.5, 2 and 2.5). The dotted value reports the debt limit distribution without imposing the zero-lower bound constraint. In all these simulations all other variables and shocks are set initially at their steady state value.

Figure 3.2 shows how different degrees of responsiveness of the risk-free interest rate R_t^F to inflation Π_t affect the debt limit distribution at the steady state. We do this by taking into account three different values of α: 1.5 (consensus), 2 (strong) and 2.5 (very strong). The sustainability of debt clearly improves with stronger responsiveness of the Taylor rule to inflation. This is due to the fact that inflation volatility falls the bigger α is, thus reducing the cost associated with deviations of inflation from the target. This, in turn, lifts profits and, with them, tax collection. Notice that the magnitude of the shift in the debt limit distribution is decreasing in α. As α increases, the distribution tends to converge to the one that would emerge if inflation was pegged to the target. Figure 3.2 also shows the difference between a model with occasionally binding ZLB constraints and a model where the ZLB is assumed to never bind. As expected, the possibility that the ZLB will bind in the future, all else equal, decreases debt sustainability at the steady state. However, the gap between the dotted line (where the ZLB is assumed to never bind) and the solid line is visible for the scenarios with the two lower α, but becomes almost negligible

FIGURE 3.3: Impact of consumption preference shocks on debt limit distributions

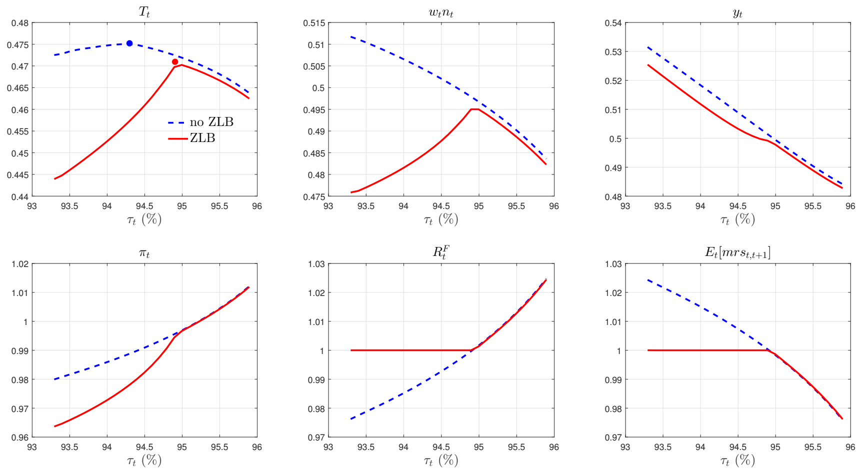


Note: The figure shows how the debt limit distributions change for simulations with an initial positive (high ν_t, black line) and negative (low ν_t, red line) preference shocks. The debt limit distribution with the preference shock at its steady-state value (mean ν_t, blue dotted line) is included for comparison. The left panel shows the debt limit distributions when the ZLB is assumed to bind while the right panel allows interest rate to take also negative values

when α reaches 2.5.

To better study the role of a binding ZLB on the debt limits we now introduce a consumption preference shocks (ν_t in equation 3.1). A positive shock reduces the marginal propensity to consume of the representative household, which results in a fall in the interest rate, output and inflation. A shock sufficiently strong can bring the economy to a situation of binding ZLB. In case of a preference shock, the interest rate channel and the indirect growth channel have opposite effects on the debt limits, as a decrease in the risk free interest rate is associated with a fall in output. When studying the debt limits at the ZLB, it is therefore of crucial importance to distinguish the role of the shock itself from its interaction with the ZLB constraint. This is done in Figure 3.3 by looking at the debt limit distributions under constrained (left panel) and unconstrained (right panel) monetary policy.

FIGURE 3.4: Binding ZLB in an economy at the debt limit



Note: The chart shows how changes in the tax rate affects the economy after the negative consumption preference shock, both in a situation in which the interest rate is left free to adjust and where the ZLB constrain hold. The first line (from left to right) of the panel report total tax revenue (the real Laffer curve), wage income and output. The second line includes inflation, the risk-free rate and the expected marginal rate of substitution.

In a situation in which interest rates are unconstrained, the negative preference shock actually improves debt sustainability. In such a situation, the reduction in interest rate that accompanies the reduction in output and inflation improves the capacity of the economy to service debt, and thus shifts the debt limit distribution to the right. In other words, the interest channel prevails over the reduction in tax collection triggered by the fall in output. Imposing the zero-lower bound constraint, however, magnifies the growth channel and mutes the interest rate channel. For the calibration used in this paper, the ZLB completely offset the benefits of the reduction in interest rates and brings the debt limit distribution close to its steady-state value.

Figure 3.4 inspects in detail the impact of a binding ZLB on debt limit determination, in order to understand the mechanism that leads to the adverse growth effect. The Laffer curve (first chart from the left in the first row) under the ZLB (red solid line) is not anymore bell-shaped as in the case of no ZBL (blue dotted line): it presents a kink in correspondence to its maximum value.

The dynamic of inflation is key to understand the shape of the Laffer curve and the dynamic of output. With a binding ZLB, monetary policy is not anymore able to stabilize inflation. The ensuing deflation increases the real interest rate depressing demand further. To clear the market, the wage rate has to decrease in order to induce households to work less. By increasing the tax rate on wage income, the fiscal authority is able to increase the marginal cost for the firms and support inflation. In turn, higher inflation lowers the real interest rate and supports private sector demand reducing the fall in wage. Actually, the ZLB offers to the government a leeway to increase the tax rate, because by doing so, it is actually able to substitute for monetary policy and reduce the distortions generated by deflation. The peak of the Laffer curve is then reached at the point in which the monetary authority is on the edge of the ZLB and inflation is at the same level that would be in place absent the constraint. Such an operation, however, is not cost free. Indeed, the tax rate that maximizes tax collection lies at the right of the no-ZLB peak. The higher tax rate puts downward pressure on labor supply, leading to a lower level of output and tax revenue. In practice, in an economy at the ZLB, fiscal policy substitutes for monetary policy: with an active use of the tax rate, the government supports inflation and through it, aggregate demand. The increase in the tax rate, however, is not harmless as it reduces output through a reduction in labor supply. This is due to the reduced disposable income received for every unit of labor supplied in the market. Despite the reduction in output, tax revenues are still substantially higher than what could be collected with tax rates below the maximizing value, because of the elevated cost of inflation fluctuations.

In synthesis, this section presented two important results: first, a more aggressive monetary policy stance can improve debt tolerance through a reduction of the price distortions in the economy; second, fiscal policy tends to replicate the work of monetary policy in an economy where the government wants to maximize tax revenues and the ZLB binds. These two considerations will be important in the simulations presented in section 3.6 .

3.5 Forward-looking fiscal space index

From a normative perspective, an important question is how much the government can use fiscal policies to stimulate the economy, without undermining the sustainability of public finances in the future. In other words, policy makers may wander what is the 'fiscal space' of the government. In this section we show how we can use fiscal limits to define a forward-looking measure of the fiscal space.

Any measure of fiscal space needs to be defined in terms of a specific fiscal instrument (i.e., tax rates, government consumption, government investment, etc.). In fact, in general equilibrium, each instrument has a different impact on the economy and, thus, on the debt-to-GDP ratio. Let F be the fiscal instrument in relation to which we want to measure the fiscal space. Let then s_t^{-F} be the state of the economy (the value taken by all exogenous shocks) at time t, with the exclusion of the value of the fiscal instrument F. For all variables other than F, the current state s_t^{-F} determines their future values, in accordance with the respective assumed data generation process.

Let B_t be government debt at time t and $\hat{B}^{\phi}(s_t)$ the debt limit corresponding to a given probability threshold ϕ conditional on the realization of the state s_t. Prior to defining the index, we first need to define an intermediate but important fiscal variable: the fiscal policy signal FS. FS indicates whether the level of debt B_t is expected to reach or go beyond the debt limit $\hat{B}_t^{\phi}$ between time t and time $t+N$. Formally, it is defined as follows:

$$FS_t = \begin{cases} 0 \text{ if } B_{t+i} < \hat{B}_{t+i}^{\phi} \quad \forall\, i \in [0, N] \\ 1 \text{ otherwise} \end{cases} \tag{3.26}$$

Let then F_t^{FS} be the level (maximum for tax rates, minimum for spending instruments) of the fiscal instrument F at which the fiscal signal FS takes the value of one given the state of the economy s_t^{-F}. [2] The fiscal space available at time t and relative to the fiscal instrument F can then be defined as

$$SP_t^F = (F_t^{FS} - F_t \,|s_t^{-F}) \tag{3.27}$$

Because of its evident non-linear nature (given the piecewise definition of the fiscal policy signal FS and of the threshold F_t^{FS}) there is no analytical solution for SP_t^F. It needs to be simulated, based on a specific set of initial shocks.

In synthesis, this fiscal index is completely forward looking and takes into account the general equilibrium implications of changes to the fiscal instrument F. Moreover, it has the interesting property of being state-contingent, as it depends on the state s_t^{-F}.

3.6 The impact and sustainability of government spending shocks

This section presents a concrete application of the tools presented above. The objective is to study the transmission mechanism of government spending shocks in light of the interaction between monetary policy, debt limits and ZLB. We perform three exercises which consider the same government spending shock under different scenarios. In the first we draw impulse-responses in normal times but under different monetary policy stance α; then, we draw impulse-responses at the ZLB, comparing the effect of the increase in government consumption with a scenario in which no fiscal stimulus is provided; finally, we repeat the first exercise, but with binding ZLB.

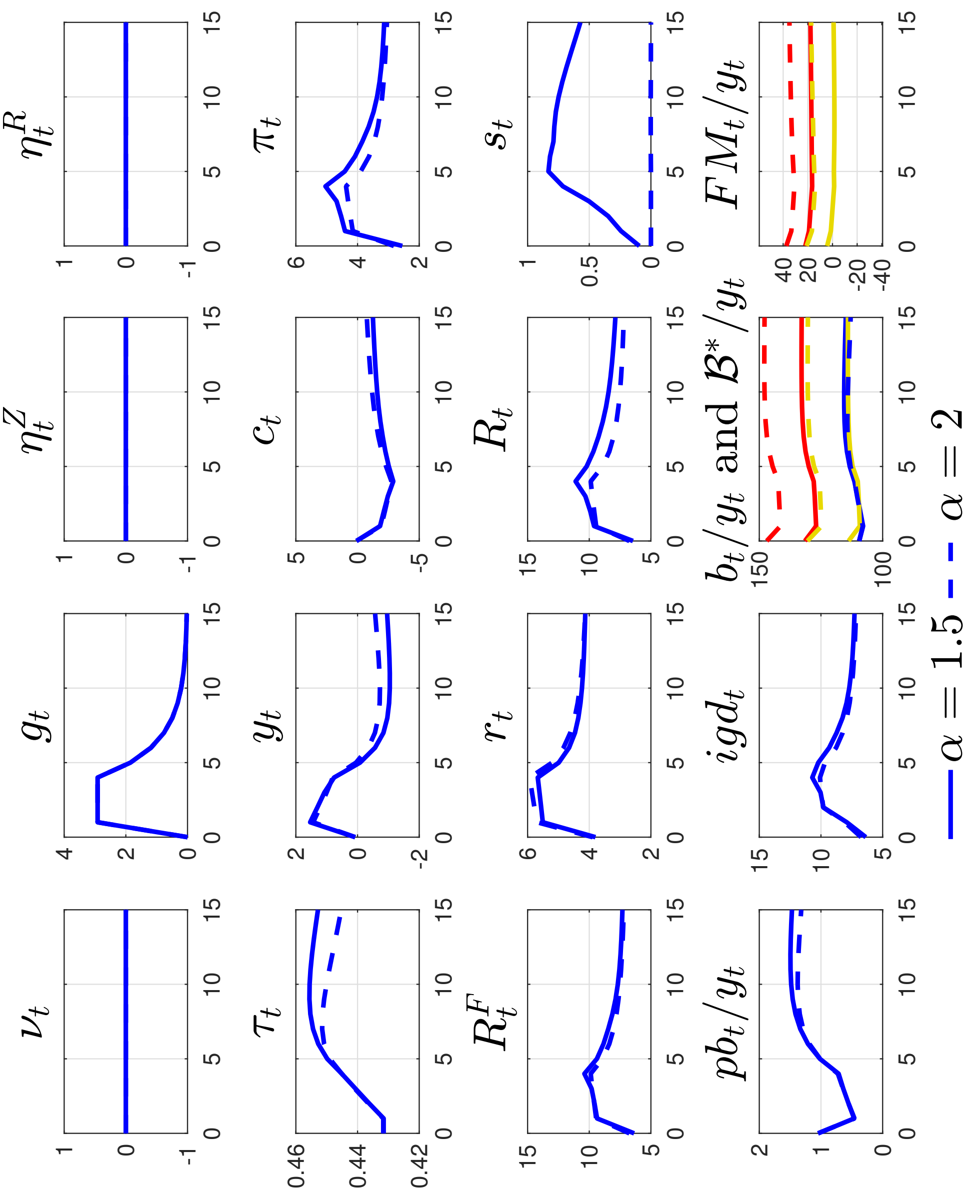
Figure 3.5 shows the effect of a positive shock to government consumption. The shock has been assumed to last 3 quarters, after which it follows an AR(1) process with a 0.85 decay parameter. To different α correspond remarkably

[2] Of course, in this section we assume that such level actually exists. This needs to be verified.

different dynamics. An increase in government consumption creates inflationary pressure to which the monetary authority responds by raising the nominal risk-free interest rate R_t^F. A more reactive monetary policy (higher α) is better able to stabilize inflation π_t, in turn, the stronger response of the real interest rate r_t dampen the response of output and consumption. Importantly, while for $\alpha = 2$ we do not observe any movement in the spread s_t, for $\alpha = 1.5$ the spread increases up to a maximum of 80 basis points after 5 quarters and then decreases slowly. This is because, as shown in figure 3.2, debt limits are much lower when $\alpha = 1.5$. Indeed, debt limits are forward looking and take into account that the monetary authority will be less able to stabilize future shocks affecting the economy. This has important consequences: the recessionary phase that follows the initial expansionary effect of the policy, is significantly more pronounced for $\alpha = 1.5$. Indeed, the increase in the spread forces the government to increase the tax rate τ_t, which, since taxes are distortionary, reduces output and crowds out private sector demand.

In figure 3.6, we study the effect of a positive shock to government consumption at the ZLB. The dotted line describes a scenario where no fiscal stimulus is provided, the continuous line describes a scenario where, as in the previous exercise, government consumption is raised of about 3 percent of steady state GDP for 3 quarters, and after that it follows an AR(1) process with a 0.85 decay parameter. Consistent with the recent literature on fiscal expansions at the ZLB, we find that the a positive shock to government consumption is particularly effective in mitigating the recession, with the drop in output y_t significantly dampened. Rather surprisingly, we find that the spending shock improves fiscal sustainability at the ZLB: the spread s_t increases up to a maximum of about 18 basis point with the shock, while it would have increased to about 30 basis point absent the shock. At the heart of the result is the fact that the increase in government expenditure is capable to mitigate significantly the drop in inflation π_t. In turn, the real interest rate r_t falls more than absent intervention. The lower real interest rate on the one hand helps to sustain demand,

FIGURE 3.5: Impact of a 3-quarters government spending shock for different α



Note: The figures represent a scenario in which spending is increased by approximately 3% of (steady-state) GDP for 3 quarters. The solid line describes the evolution of the economy for $\alpha = 1.5$, the dotted line describes the evolution of the economy for $\alpha = 2$. In the bottom right panel, the red line indicates the evolution of the 30% debt limit threshold, i.e., the value on the debt limit distribution that corresponds to a 30% default probability.

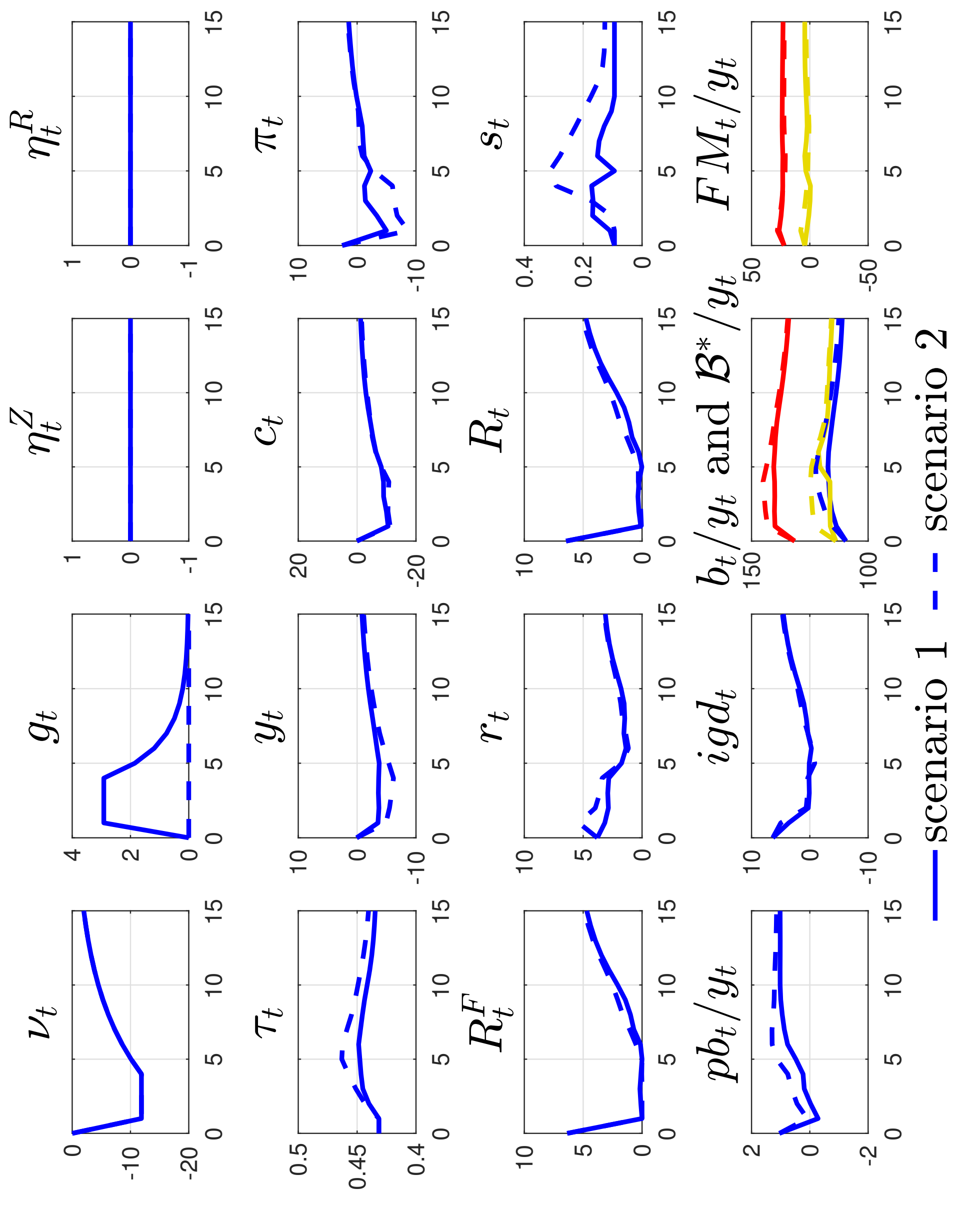
on the other hand helps the government to finance public expenditure, without significantly affect the debt level. Indeed, despite the government runs a lower primary balance pb/y as compared to a situation without shock, debt increases less than without intervention. In turn, this implies that the government can afford a lower tax rate τ with positive effects on output.

In figure 3.7, we study the same shock considered in 3.6; however, here we compare the dynamics that arise respectively with $\alpha = 1.5$ (the case considered in 3.6) and $\alpha = 2$. The two dynamics are remarkably different. For $\alpha = 2$, the fact that a more reactive monetary authority is better able to peg expectations to the target, implies that the deflation following the preference shock that leads to the ZLB is significantly lower than with $\alpha = 1.5$. As a consequence, the real interest rate r_t on impact falls much more, helping to stabilize the economy. Despite the primary balance to GDP, pb/y, decreases substantially to finance the increase in public expenditure, the stock of debt will actually decrease over time due to the low interest rate environment. Because of that, the government is able to decrease marginally the tax rate τ stimulating the demand further. Both output and consumption fall significantly less. Notice that debt limits actually increase (both the dashed yellow and red line on the last quadrant), which results in the absence of spread s_t.

3.7 Conclusions

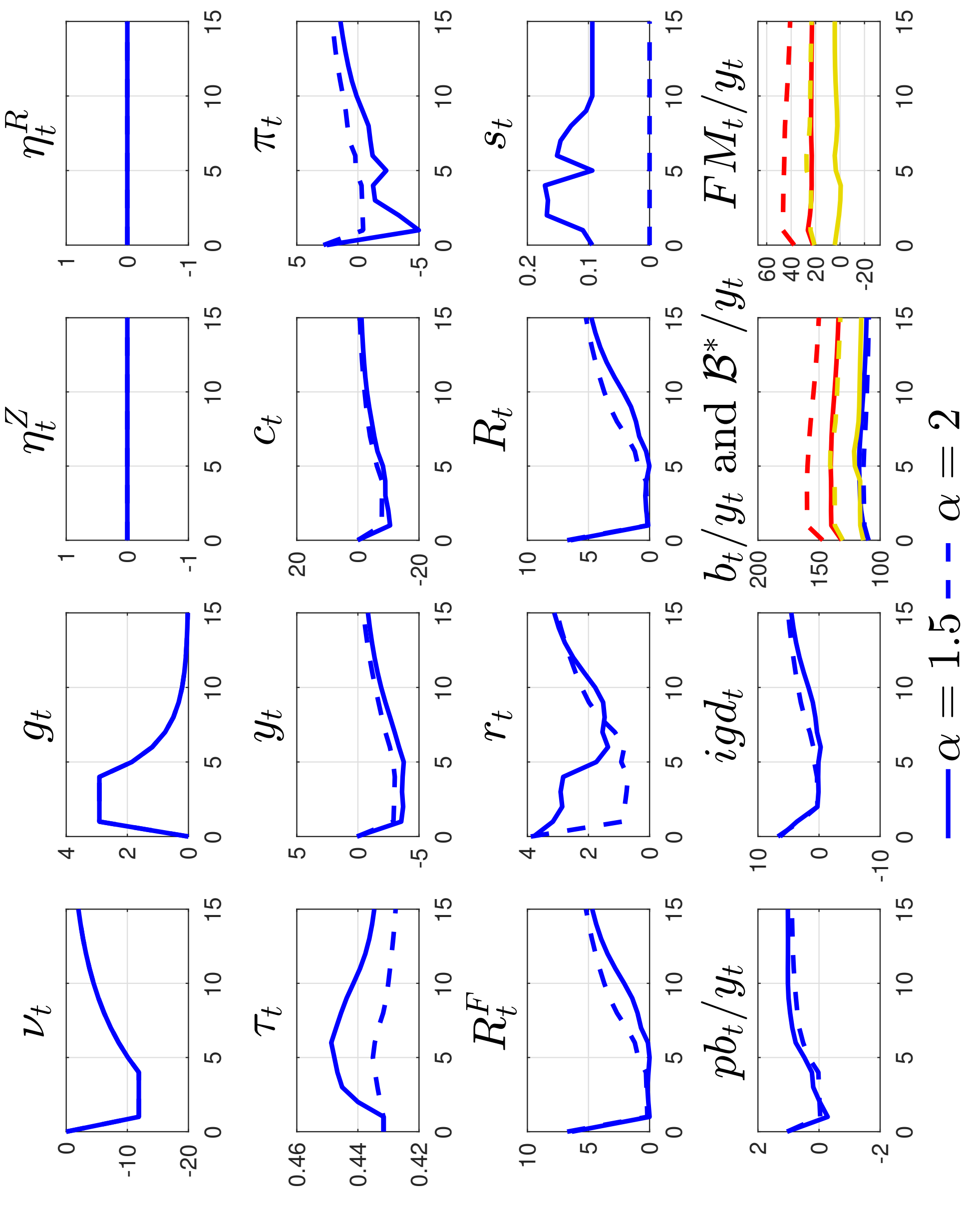
We analyzed fiscal sustainability and monetary fiscal policy interactions through the lens of a DSGE model with risky sovereign debt à la Bi (2012). We found that a looser monetary policy stance implies larger inflation fluctuations that, by decreasing output and the tax revenue, negatively affect fiscal sustainability. In normal times this reduces the fiscal space available to increase government consumption. Indeed, by engaging in expansionary policies, the government risks to increase spreads that might reverse the initial stimulative effect of the policy.

FIGURE 3.6: Impact of a 3-quarters government spending shock in presence of binding ZLB



Note: The figures represent a scenario in which the ZLB is binding due to a persistent preference shock and $\alpha = 1.5$. The solid line describes the evolution of the economy when spending is increased by approximately 3% of (steady-state) GDP for 3 quarters. The dotted line describes the evolution of the economy absent shock to government consumption. The bottom right panel indicates the evolution of the fiscal margin corresponding to, respectively, a 30%(red line) and 0.4%(yellow line) default probability threshold .

FIGURE 3.7: Impact of a 3-quarters government spending shock at the ZLB for different α.



Note: The figures represent a scenario in which the ZLB is binding due to a persistent preference shock and spending is increased by approximately 3% of (steady-state) GDP for 3 quarters. The solid line describes the evolution of the economy for $\alpha = 1.5$. The dotted line describe the evolution of the economy for $\alpha = 2$. The bottom right panel indicates the evolution of the fiscal margin corresponding to, respectively, a 30%(red line) and 0.4%(yellow line) default probability threshold .

Consistent with previous findings, at the ZLB an increase in government expenditure is effective in reducing the fall in output and consumption. Notably, it does so without affecting the sustainability of debt, which actually improves in our calibration. By creating inflationary pressure, an increase in government consumption is able to reduce the real interest rate. In turn, this relaxes the ZLB constraint and reduces the cost of servicing the debt. However, the effectiveness of the policy largely depends on the underlying monetary policy stance. A more responsive monetary policy in normal times is better able to peg inflation expectation to the target. In turn, this dampens the deflationary pressure during periods of binding ZLB, allowing the real interest rates to fall further down, this way sustaining aggregate demand and increasing the fiscal space for the government.

Appendix A

A.1 Proof of Proposition 3

1. Proof of 1. The Hamiltonian-Jacobi-Bellman equation associated with the investors' problem in (1.9) is;

$$(r+\lambda)V(a,q) = \max_{\dot{a}} \left[-q(\dot{a}+\delta a) + (r+\delta+\lambda)a + V_a'\dot{a} + V_q'\dot{q}\right],$$

where we have dropped the time indexes for simplicity of notation. Notice that the assumption of perfect competition and the fact that investors are atomistic implies that $V(a,q) = \tilde{V}(q)a$, which means that the unit value of an asset must be independent of the quantity of asset holdings. Then, we have:

$$(r+\lambda)\tilde{V}(q) = \max_{\dot{a}} \left[-(q-\tilde{V}(q))\frac{\dot{a}}{a} + (r+\delta(1-q)+\lambda) + \tilde{V}_q'\dot{q}\right].$$

If $q > \tilde{V}(q)$, the price of the asset would be larger than its value and the investors would like to sell an arbitrarily large number of assets. Viceversa, if $q < \tilde{V}(q)$, the price of the asset would be lower than its value and the investors would demand an infinite number of assets. It follows that in equilibrium it must be that $q = \tilde{V}(q)$. Substituting this relationship in the above expression we obtain statement 1 of the Proposition.

2. Proof of 2. The value of a bond one instant before default is

$$\begin{aligned} q(T-dt) &= \int_{T-dt}^{T} (r+\delta+\lambda)e^{-(r+\delta+\lambda)(s-T+dt)}ds + \int_{T}^{\infty} \phi\lambda e^{-(r+\lambda)(s-T)}ds \\ &= 1 - e^{-(r+\delta+\lambda)dt} + \frac{\lambda\phi}{r+\lambda} \end{aligned}$$

taking the limit for $dt \to 0$ we get $q(T) = \frac{\lambda\phi}{r+\lambda}$.

3. Proof of 3. Since it is never optimal for the government to default in the high income state, the value of a bond solves

$$q(t) = \int_t^\infty (r+\delta)e^{-(r+\delta)(s-t)}ds$$

and $q(t) = 1, \forall t \geq T^j$.

A.2 Derivation of the continuous time Euler Equation in equation (1.18)

Define the current value Hamiltonian:

$$H(b,p,c,t) = u(c(t)) + \lambda W_b^j(b(t)) + p(t)\dot{b}(t),$$

where $p(t)$ is the costate variable, $b(t)$ is the state variable, $c(t)$ is the control variable and $u(c)$ is a generic utility function which satisfies Inada conditions. The first order conditions of the optimal control problem are

$$\begin{aligned} H_c &= 0, \\ -H_b &= \dot{p}(t) - (\rho+\lambda)p(t), \\ H_p &= \dot{b}(t). \end{aligned}$$

Substituting the derivatives of the Hamiltonian

$$\begin{aligned} q(t)u'(c(t)) &= p(t), \\ \lambda W_b^j(b(t)) + p(t)\left[\frac{\rho+\delta}{q(t)} - \delta\right] &= -\dot{p}(t) + (\rho+\lambda)p(t), \\ \dot{b}(t) &= \frac{1}{q(t)}\left(y_L - c(t) + (\rho+\delta)b(t)\right) - \delta b(t), \end{aligned}$$

and consolidating the first two equations:

$$\begin{aligned} \lambda W_b^j(b(t)) + (\rho+\delta)u'(c(t)) - \delta q(t)u'(c(t)) = &-\dot{q}(t)u'(c(t)) - q(t)u''(c(t))\dot{c}(t) + \\ &+ (\rho+\lambda)q(t)u'(c(t)). \end{aligned} \tag{A.1}$$

Substituting for $\dot{q}(t) = (q(t)-1)(\rho+\delta+\lambda)$ and using the fact that with log-utility $-\frac{u''(c(t))c(t)}{u'(c(t))} = 1$ and $u'(c(t)) = \frac{1}{c}$ we obtain (1.18)

A.3 Derivation of the Terminal conditions in equation (1.19)-(1.21)

Problem (1.14)-(1.17) requires a simultaneous determination of optimal control and the terminal time. These problems are usually called *free terminal time problems* and it is well known that optimality for the terminal time requires an additional transversality condition. Let T be the terminal time and $S(b(T), T, t)$ denote the *salvage value function*:

$$S(b(T), T, t) \equiv W^d(b(T))e^{-(\lambda+r)(T-t)}.$$

At the optimum terminal time, T, the costate variable must satisfy:

$$p(T) = S_b(b(T), T, t),$$

while the transversality condition is given by

$$H(b(T), p(T), c(T), T) + S_T(b(T), T, t) = 0.$$

The transversality condition requires that at the optimal terminal time, the benefit of delaying default of one instant, given by the Hamiltonian evaluated at T, is equal to opportunity cost of delaying default, given by the derivative of the salvage function with respect to T. Together with the budget constraint, the terminal conditions of the problem define a system of three equations:

$$\begin{aligned}
\log(c(T)) + \lambda W^j(b(T)) + p(T)\dot{b}(T) &= (\rho+\lambda)W^d(b(T)) \\
p(T) &= W^d_b(b(T)), \\
\dot{b}(T) &= \frac{1}{q(T)}\left[y_L - c(T) + (\rho+\delta)b(T)\right] - \delta b(T).
\end{aligned}$$

Using the fact that $p(T) = \frac{q(T)}{c(T)}$ and using (1.12), we get

$$\begin{aligned}
\log(c(T)) - \log(y_L) &= \lambda\left[W^j(\phi b(T)) - W^j(b(T))\right] - W^d_b(b(T)\dot{b}(T) \\
\frac{q(T)}{c(T)} &= W^d_b(b(T)) \\
\dot{b}(T) &= \frac{1}{q(T)}\left[y_L - c(T) + (\rho+\delta)b(T)\right] - \delta b(T).
\end{aligned}$$

By substituting $W_b^d(b(T))$ from equation (1.12), $W_b^j(b(T))$ from equation (1.11) and $q(T)$ in (1.17), the system of equations simplifies to

$$\log(c(T)) - \log(y_L) = \lambda \left[W^j(\phi b(T)) - W^j(b(T)) \right] - W_b^d(b(T))\dot{b}(T)$$
$$c(T) = y_H + r\phi b(T),$$
$$\dot{b}(T) = \frac{\rho + \lambda}{\lambda \phi} \left[y_L - c(T) + (\rho + \delta)b(T) \right] - \delta b(T).$$

A.4 Proof of Proposition 4

Equations (1.26)-(1.28), define a system of three equations in three unknowns, $\dot{b}, q^P, c$, given b. At the steady state $\dot{b} = 0$ the system simplifies to:

$$\begin{cases} c = y_L \left(\frac{y_H + \rho\phi b}{y_H + \rho b} \right)^{\frac{\lambda}{r}} \\ q^P = \frac{\phi\lambda}{\rho+\lambda} \left(\frac{c}{y_H + \rho\phi b} \right) \\ c - y_L = b \left[\rho + \delta(1 - q^P) \right] \end{cases}$$

Unfortunately, as standard for a system of non-linear equations, we cannot prove the existence of a steady state and, therefore, we have to rely upon numerical solutions. However, provided that a steady state does exist, we can still study its stability properties. Equations (1.26)-(1.28), define an autonomous system of three equations in three unknowns, $\dot{b}, q^P, c$, given b. We use the notation $W_b^j(\cdot)$ and $W_{bb}^j(\cdot)$ to denote respectively the first and the second derivative with respect to b of the function $W^j(\cdot)$. Taking derivatives of each equation in the system (1.26)-(1.28) with respect to b we obtain

$$\underbrace{\begin{bmatrix} \frac{1}{c} & \frac{\lambda}{\rho+\lambda} W_b^j(\phi b) & 0 \\ -\frac{\lambda}{\rho+\lambda} W_b^j(\phi b) & 0 & 1 \\ 1 & q^P & \dot{b} + \delta b \end{bmatrix}}_{\equiv A} \begin{bmatrix} \frac{\partial c}{\partial b} \\ \frac{\partial \dot{b}}{\partial b} \\ \frac{\partial q^P}{\partial b} \end{bmatrix} = \underbrace{\begin{bmatrix} \lambda \left(W_b^j(\phi b) - W_b^j(b) \right) - \frac{\lambda}{\rho+\lambda} W_{bb}^j(\phi b)\dot{b} \\ \frac{\lambda}{\rho+\lambda} W_{bb}^j(\phi b) c \\ \rho + \delta(1 - q^P) \end{bmatrix}}_{\equiv v}. \tag{A.2}$$

The determinant of A is

$$det(A) = -\frac{q^P}{c} + \frac{\lambda}{\rho+\lambda} W_b^j(\phi b) \left(1 + \frac{\lambda}{\rho+\lambda} W_b^j(\phi b)(\dot{b} + \delta b) \right)$$
$$= \left(\frac{\lambda}{\rho+\lambda} W_b^j(\phi b) \right)^2 (\dot{b} + \delta b).$$

where the second equality is obtained substituting for equation (1.27). Let A_2 be the matrix obtained by substituting the second column in A with the vector v, the determinant of A_2 reads

$$det(A_2) = \left(\frac{\lambda}{\rho+\lambda}W^j_{bb}(\phi b)(\dot{b}+\delta b) - (r+\delta(1-q^p))\frac{1}{c}\right) + \\ + \left(\lambda\left(W^j_b(\phi b) - W^j_b(b)\right) - \frac{\lambda}{\rho+\lambda}W^j_{bb}(\phi b)\dot{b}\right)\left(1+\frac{\lambda}{\rho+\lambda}W^j_b(\phi b)(\dot{b}+\delta b)\right)$$

By Cramer rule, $\frac{\partial \dot{b}}{\partial b} = \frac{det(A_2)}{det(A)}$. A steady state is stable if and only if the derivative $\frac{\partial \dot{b}}{\partial b}$ evaluated at the steady state is negative, formally: $\frac{\partial \dot{b}}{\partial b}|_{\dot{b}=0} < 0$. Notice that at the steady state $\dot{b} = 0$, and, therefore, $det(A) < 0$ since we are restricting our domain of interest on $b < 0$. Stability follows if we can show that at the steady state $det(A_2|\dot{b} = 0) > 0$.

$$det(A_2|\dot{b}=0) = -\lambda\left(W^j_b(b) - W^j_b(\phi b)\right) - (r+\delta(1-q^P))\frac{1}{c} + \\ + \frac{\lambda}{\rho+\lambda}W^j_{bb}(\phi b)\delta b + \frac{\lambda^2}{\rho+\lambda}\left(W^j_b(\phi b) - W^j_b(b)\right)W^j_b(\phi b)\delta b. \tag{A.3}$$

The envelope condition associated to the government problem which can be derived by taking derivatives with respect to b of the Hamiltonian-Jacobian-Bellman equation in 1.26, reads

$$\left(W^d_{bb} - \frac{q^{P\prime}(b)}{q^P(b)}\right)\dot{b} = -\frac{W^d_b}{q}\left[\rho+\delta(1-q^P)\right] - \lambda\left(W^j_b(\phi b) - W^j_b(b)\right), \tag{A.4}$$

and implies that at the steady state

$$\frac{W^d_b}{q}\left[\rho+\delta(1-q^P)\right] - \lambda\left(W^j_b(\phi b) - W^j_b(b)\right) = 0$$

Substituting the FOCs of the planner problem, $\frac{q^P(t)}{c(t)} = W^d_b$, it follows that we can simplify $det(A_2|\dot{b} = 0)$ to

$$det(A_2|\dot{b}=0) = \frac{\lambda}{\rho+\lambda}W^j_{bb}(\phi b)\delta b + \frac{\lambda^2}{\rho+\lambda}\left(W^j_b(\phi b) - W^j_b(b)\right)W^j_b(\phi b)\delta b > 0.$$

The sign follows immediately from the fact that $W^j_{bb}(\phi b) < 0$, $\left(W^j_b(\phi b) - W^j_b(b)\right) < 0$, $W^j_b(\phi b) > 0$ and $b < 0$. This proves that if a steady state does exist, it must be stable. In addition, since the inequality $\frac{\partial \dot{b}}{\partial b}|_{\dot{b}=0} > 0$ is satisfied for any possible steady state in the domain $b < 0$, it must be the case that if a steady state exists it must also be unique on this domain. From A.4, we must also have that on the domain of interest where $\dot{b} < 0$, $\frac{q^{P\prime}(b)}{q^P(b)} < 0$. That because the RHS of A.4 is negative for every q^P lower than the steady state level, and in order for the LHS to be negative, since $W^d_{bb} < 0$, it must be $\frac{q^{P\prime}(b)}{q^P(b)} < 0$. Hence, the policy price is decreasing in b, or equivalently, increasing in t.

A.5 Proof of Proposition 5

1. Proof of 1.

   At the time of intervention T, $q^P(b(T)) = \frac{\phi\lambda}{\rho+\lambda}$. Therefore, the government budget constraint in equation (1.28) reads

   $$\frac{\phi\lambda}{\rho+\lambda}\dot{b}(T) = y_L - c(T) + \left(\rho + \delta\left(1 - \frac{\phi\lambda}{\rho+\lambda}\right)\right)b(T).$$

   Replace the equation above into the last term of the ICC in equation (1.31) and evaluate the ICC at T:

   $$y_L - c(T) - \lambda(1-\phi)b(T) - \left[y_L - c(T) + \left(\rho + \delta\left(1 - \frac{\phi\lambda}{\rho+\lambda}\right)\right)b(T)\right].$$

   Simplifying, it becomes:

   $$-\left[\lambda(1-\phi) + \left(\rho + \delta\left(1 - \frac{\phi\lambda}{\rho+\lambda}\right)\right)\right]b(T) > 0.$$

   Since all the coefficients are positive, and ϕ and λ are less then one, the term in square bracket is positive. Therefore, the ICC at time T is satisfied whenever the government defaults with some debt $b(T) < 0$.

2. Proof of 2.

   Denote $\bar{b}$ the steady state level of debt, such that $\dot{b}(\bar{b}) = 0$. Provided that a steady state exists, if the intervention is incentive compatible ($ICC > 0$) at the steady state, then the intervention will continue until the jump to the high income state. We will show that this cannot be the case. The system of equations (1.26)-(1.28) at the steady state reads:

   $$\begin{cases} c = y_L\left(\frac{y_H+\rho\phi\bar{b}}{y_H+\rho\bar{b}}\right)^{\frac{\lambda}{\rho}}, \\ q^P = \frac{\phi\lambda}{\rho+\lambda}\left(\frac{c}{y_H+\rho\phi\bar{b}}\right), \\ c - y_L = \bar{b}[\rho + \delta(1-q^P)]. \end{cases}$$

   This proof consists of two parts.

   Part 1. First we show that at the steady state $\bar{b}$, it must be $\bar{b} < \frac{y_L-y_H}{\rho}$. Define ϵ such that

$\bar{b} = \frac{y_L - y_H}{\rho} + \epsilon$, we can then rearrange that expression as

$$y_H + \rho\bar{b} = y_L + \rho\epsilon$$

or equivalently,

$$y_H + \rho\phi\bar{b} = [\phi y_L + (1-\phi)y_H] + \phi\rho\epsilon.$$

Define the variable ζ as:

$$\zeta \equiv \frac{y_H + \rho\phi\bar{b}}{y_H + \rho\bar{b}} = \frac{[\phi y_L + (1-\phi)y_H] + \phi\rho\epsilon}{y_L + \rho\epsilon}.$$

Notice that $\zeta > 1$ since $\phi \leq 1$ and $\bar{b} < 0$. We can now restate the system in terms of ζ and ϵ as

$$\begin{cases} c = y_L \zeta^{\frac{\lambda}{\rho}}, \\ q^P = \left(\frac{\phi\lambda}{\rho+\lambda}\right) \frac{y_L \zeta^{\frac{\lambda}{\rho}}}{[\phi y_L + (1-\phi)y_H] + \phi\rho\epsilon} \\ y_L \zeta^{\frac{\lambda}{\rho}} - y_L = \left(\frac{y_L - y_H + \epsilon\rho}{\rho}\right)\left(\rho + \delta(1-q^P)\right). \end{cases}$$

By substituting q^P in the third equation:

$$\begin{aligned} \zeta^{\frac{\lambda}{\rho}} \left[y_L + \left(\frac{y_L - y_H + \epsilon\rho}{\rho}\right) \frac{\delta\phi\lambda y_L}{(\rho+\lambda)\left[(\phi y_L + (1-\phi)y_H) + \phi\rho\epsilon\right]} \right] = \\ = y_L + \left(\frac{y_L - y_H + \epsilon\rho}{\rho}\right)(\rho + \delta). \end{aligned}$$

Now, $\zeta > 1$ and $y_L + \rho\epsilon < y_H$, therefore a necessary condition for the equality to be satisfied is that

$$\frac{\delta\phi\lambda y_L}{(\rho+\lambda)\left[(\phi y_L + (1-\phi)y_H) + \phi\rho\epsilon\right]} > \delta + \rho,$$

rearranging the inequality

$$-\frac{\rho\phi(\delta+\rho+\lambda)}{\rho+\lambda} y_L - (\delta+\rho)(1-\phi)y_H > \phi\rho\epsilon.$$

which implies $\epsilon < 0$.

Part 2. We can write the first equation of the system above in logs as:

$$ln(c) - ln(y_L) = \frac{\lambda}{\rho}\left[ln(y_H + \rho\phi\bar{b}) - ln(y_H + \rho\bar{b})\right].$$

From part 1, at the steady state $c \geq y_L$ which implies, from the government budget constraint evaluated at the steady state (third equation of the system), that $q^P \geq \frac{\delta+\rho}{\delta}$. Then

$$c > y_H + \rho\phi\bar{b}.$$

Moreover, from part 1, we have

$$y_L > y_H + \rho\bar{b}.$$

By strict concavity of the logarithmic function (the result is proved in Lemma 7 below), we have:

$$\frac{ln(c) - ln(y_L)}{c - y_L} < \left[\frac{ln(y_H + \rho\phi\bar{b}) - ln(y_H + \rho\bar{b})}{\rho(\phi - 1)\bar{b}}\right].$$

Substituting $ln(c) - ln(y_L) = \frac{\lambda}{\rho}[ln(y_H + \rho\phi\bar{b}) - ln(y_H + \rho\bar{b})]$, we have:

$$c - y_L > \lambda(\phi - 1)\bar{b}.$$

Hence, the ICC is negative at the steady state. Intuitively, the cost of avoiding default, $c - y_L$ is higher than the benefit for the investors $\lambda(\phi - 1)\bar{b}$.

Lemma 7. *Let $C \rightarrow R$ be an open interval, $f : C \rightarrow R$ is concave iff for any $a, b, c, d \in C$, with $a < b < c < d$,*

$$\frac{f(c) - f(a)}{c - a} \geq \frac{f(d) - f(b)}{d - b}.$$

Proof. . We first show that:

$$\frac{f(c) - f(a)}{c - a} \geq \frac{f(d) - f(a)}{d - a}.$$

Suppose that f is concave and take any $a, b, c, d \in C$, $a < b < c < d$. Since $(c - a) > 0$ and $(d - a) > 0$, the expression above holds iff:

$$\frac{f(c) - f(a)}{c - a} \geq \frac{f(d) - f(a)}{d - a},$$

which holds iff (collecting terms in $f(c)$),

$$f(c) \geq \left(1 - \frac{c-a}{d-a}\right) f(a) + \left(\frac{c-a}{d-a}\right) f(d).$$

Since f is concave, the latter holds taking $\theta = \left(\frac{c-a}{d-a}\right) \in (0,1)$. Moreover, verifying that $c = (1-\theta)a + (\theta)d$, any function that satisfies the equation needs indeed to be concave.

$$f(\theta d + (1-\theta)a) \geq \theta f(d) + (1-\theta) f(a).$$

Similarly we can show that:

$$\frac{f(d) - f(a)}{d-a} \geq \frac{f(d) - f(b)}{d-b}.$$

Collecting terms in $f(b)$,

$$f(b) \geq \left(1 - \frac{d-b}{d-a}\right) f(d) + \left(\frac{d-b}{d-a}\right) f(a).$$

The previous proof goes through, taking $\theta = \left(\frac{d-b}{d-a}\right)$, and verifying that, indeed, $b = (1-\theta)d + (\theta)a$. □

A.6 Proof of Proposition 6

1. Suppose $q(T^P|\alpha) > q^P(T^P)$, then, by continuity of q and q^P, the government would be better off keep borrowing from the market and delay the intervention. This way it can relax its budget constraint: by borrowing at an higher price, it can finance the same $\dot{b}$ but consume more.

2. Suppose $q(T^P|\alpha) > q^P(T^P)$. Then the government would had been better to anticipate the intervention. The argument is symmetric to the one presented above.

A.7 Proof of Proposition 7

1. Part 1.

   From 1.34, we have that

$$\frac{\partial}{\partial \alpha} \Delta V(\alpha, 0) = (1-\alpha) \frac{\partial}{\partial \alpha} \int_{T^P}^{T^E} -G(s) e^{-(r+\lambda)(T^E - t)} ds + \int_{T^P}^{T^E} G(s) e^{-(r+\lambda)(T^E - t)} ds$$

Since $G(t)$ is negative $\forall t \geq T^P$, the second term is positive. For the first term to be positive for $\alpha > 1$ we need to show that

$$\frac{\partial}{\partial \alpha} \int_{T^P}^{T^E} -G(s) e^{-(r+\lambda)(T^E - t)} ds < 0.$$

We use a perturbation argument. Suppose that we start from an equilibrium, where $b(T^P|\alpha)$ is debt at intervention and $b(T^E)$ is debt at default. First, notice that $b(T^E)$ is set by the policy-maker independently of α (the ICC does not depend on the fiscal rule). On the other hand, by altering the bond price, α affects the equilibrium debt at intervention. Consider a marginal increase in α, keeping intervention fixed at the initial equilibrium $b(T^P|\alpha)$. Increasing α shifts down the market bond price in 1.33, hence $q(b(T^P|\alpha)|\alpha + d\alpha) < q^P(b(T^P)|\alpha)$, $\forall d\alpha > 0$. By proposition 6, it cannot be optimal for the government to stop at $b(T^P|\alpha)$, better to stop one instant before. Therefore, it must be $b(T^P|\alpha + d\alpha) > b(T^P|\alpha)$ and $(T^E - T^P|\alpha + d\alpha) > (T^E - T^P|\alpha)$. Also, $G(t)$ is decreasing in α since for any given $\dot{b}(b(t))$, the distance $q^P(b(t)) - q(b(t)|\alpha + d\alpha) > q^P(b(t)) - q(b(t)|\alpha)$ is increasing $\forall b(t) \in [b(T^P|\alpha), b(T^E)]$. It follows that the derivative has a negative sign.

2. Part 2.

   For any $b(T^P)$ and $d\alpha > 0$, $q(b(T^P)|\alpha) - q(b(T^P)|\alpha + d\alpha) > 0$. An higher α restricts the inter-temporal budget constraint of the government. To sustain any given borrowing plan $\{\dot{b}(t)\}_{t=0}^{T^P}$, the government will have to consume less. It follows that the welfare of the government should be monotonically decreasing in α.

A.8 Proof of Proposition 8

1. Let $\bar{\alpha}$ be such that $q(T^P|\bar{\alpha}) = q(T)$, hence by the terminal conditions of the government also $b(T^P|\bar{\alpha}) = b(T)$. It follows that, given that the government will face the same bond price, the dynamic of the economy pre-intervention is exactly the same with and without policy, hence $\Delta W(\bar{\alpha}, 0) = 0$. Since intervention has positive net present value over default ($ICC > 0 \; \forall t \in [T^P, T^E]$), it must be $\Delta V(\bar{\alpha}, 0) > 0$. By equation 1.34, $\bar{\alpha} > 1$.
2. Let $\alpha = 1$, by equation 1.34 it follows immediately that $\Delta V(1, 0) = 0$. By proposition 7, Moreover $\Delta W(1, 0) > 0$ since by proposition 7 $\Delta W(\alpha, 0)$ is monotonically decreasing in α and from above we know that $\Delta W(\bar{\alpha}, 0) = 0$ for $\bar{\alpha} < 1$.
3. By proposition (7) it follows immediately that the Pareto set is identified by $\alpha \in [1, \bar{\alpha}]$.

Appendix B

B.1 Solution algorithm

Before calculating the debt limit distribution via Monte Carlo simulations, the absence of functional forms for all the endogenous variables requires the solution of part of the non-linear model (i.e., for the real wage, the inflation rate and the maximum tax rate), with numerical methods. The solution is based on the monotone mapping method, developed by Coleman, 1991 and Davig, 2004, which discretizes the state space and conjectures candidate decision rules that reduce the system to a set of first-order expectational difference equations. The decision rules for the real wage $w_t^* = f^w(\psi_t^1)$, the inflation rate $\pi_t^* = f^\pi(\psi_t^1)$ and the corresponding maximum tax rate $\tau_t^* = f^\tau(\psi_t^1)$ are solved in the following steps.

1. Discretize the state space $\psi_t^1 = \{A_t, \nu_t, G_t, \eta^z, \eta^R\}$ [1]

2. For $i = 1, 2, ...$, make a guess for the decision rules $(f_g^w, f_g^\pi, f_g^\tau)$ over the state space. If $i = 1$, set $(f_g^w, f_g^\pi, f_g^\tau)$ to their steady state values; if $i > 1$, set $(f_g^w, f_g^\pi, f_g^\tau)$ to the solutions in the previous iteration $(f_{i-1}^w, f_{i-1}^\pi, f_{i-1}^\tau)$.

3. At each grid point, solve the model and obtain the updated rule $(f_i^w, f_i^\pi, f_i^\tau)$ using the given rule $(f_{i-1}^w, f_{i-1}^\pi, f_{i-1}^\tau)$ as a guess. Given equations 3.4, 3.16, 3.22 and 3.24 to pin down (n_t, Y_t, R_t^F, c_t), respectively, use the model equations 3.5, 3.15 and 3.21 to solve the non-linear model and determine the decision rules $(f_i^w, f_i^\pi, f_i^\tau)$. In particular, maximize 3.21 subject to the non-linear constraints 3.5 and 3.15 and non-negativity constraints on endogenous variables as appropriate. The integrals implied by the expectation terms are evaluated using numerical quadrature. The exogenous AR(1) processes are approximated as first-order Markov processes according to the quadrature approach by Tauchen and Hussey, 1991.

4. Notice that $(f_{i-1}^w, f_{i-1}^\pi, f_{i-1}^\tau)$ are assumed to be decision rules at t+1 when evaluating expectations, as they provide a set of intra-temporally consistent solutions for the optimising agents. To ensure that the solution is also inter-temporally consistent, establish

[1] The number of grid points and state variables actually considered can vary depeding on the problem at hand, in order to deal with curse of dimensionality

a rule to check convergence of the decision rules $(f_i^w, f_i^\pi, f_i^\tau)$ and $(f_{i-1}^w, f_{i-1}^\pi, f_{i-1}^\tau)$ as follows:

(a) if $\max\{|(f_i^w, f_i^\pi, f_i^\tau) - (f_{i-1}^w, f_{i-1}^\pi, f_{i-1}^\tau)| > 1e-6\}$, go back to step 2;

(b) otherwise, $(f_i^w, f_i^\pi, f_i^\tau)$ are the decision rules.

To solve the full model, the same algorithm is used, but with an enlarged state space that embeds the stock of debt b_t as a state variable.

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